Price Competition: Bertrand Model:

Both firms strategize on the price level that they will charge.

Q. Suppose mkt demand curve: $P = 10 - q$, $C_1(q_1) = 2q_1$ $C_2(q_2) = 2q_2$.
Both firms compete through Prices.

(i) Find the Bertrand prices: (equillibrium prices)

$MC_1 = 2$, $MC_2 = 2$. [Lowest possible prices that the firm can set because if $P < MC$, then the firm incurs loss].

(i) If $P_1 < P_2 \Rightarrow q_2 = 0$, $q_1 = (10 - P_1)$ ... [Firm I has entire mkt share].

(ii) If $P_2 < P_1 \Rightarrow q_1 = 0$, $q_2 = (10 - P_2)$ ... [Firm II has entire mkt share].

(iii) If $P_1 = P_2 = P \Rightarrow q_1 = q_2 = \frac{(10 - P)}{2}$ .. [Both firms have equal share of the mkt].

$$q_1^* = \begin{cases} (10 - P_1), & P_1 < P_2 \\ \frac{1}{2}(10 - P_1), & P_1 = P_2 \\ 0, & P_1 > P_2 \end{cases} \qquad q_2^* = \begin{cases} (10 - P_2), & P_2 < P_1 \\ \frac{1}{2}(10 - P_2), & P_2 = P_1 \\ 0, & P_2 > P_1 \end{cases}$$

Begin with $P_1 < P_2$ ---- Firm I sells at a cheaper rate. & $q_2 = 0 \Rightarrow \pi_2 = 0$. Firm II will undercut/reduce its price to slightly less than P_1, i.e $P_2 - \varepsilon < P_1$. Now $q_1 = 0 \Rightarrow \pi_1 = 0$. Now Firm I will also undercut to earn positive profit. Hence both firms will continue to undercut prices till they reach the lowest possible price that they can set. Hence, $\boxed{p_1^* = p_2^* = MC}$ is the Bertrand Equillibrium. [Bertrand Paradox].

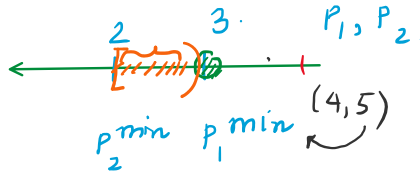
$$p_1^* = p_2^* = 2 \quad \ldots\ldots \text{[Bertrand Equilibrium]}.$$

(ii) If $c_1(q_1) = 3q_1$, $c_2(q_2) = 2q_2$. Find Bertrand Prices.

$MC_1 = 3 > MC_2 = 2$. $\qquad p_1^{min} = 3$, $p_2^{min} = 2$.

For $(p_1, p_2) > (3,3)$ both firms will undercut.

$\therefore$ Any $(p_1, p_2) > (3,3)$ will not be equillibrium.



At (3,3), $\Rightarrow$ $p_1 = 3 = MC_1$ ... [Firm I will not be able to undercut any further]

$\therefore$ $p_1^* = 3$, $\pi_1^* = 0$.

If $p_2 = 3$, Total demand $q = 10 - p = 7 \Rightarrow q_1^* = q_2^* = \frac{7}{2}$.

$$\pi_2 = 3 \times \frac{7}{2} - 2 \times \frac{7}{2} = \frac{7}{2} = 3.5$$

$\therefore$ At $(p_1, p_2) = (3, 3)$, $\pi_2 = 7/2$. We will evaluate if undercutting price by Firm II will be more beneficial or not. [i.e $\pi_2 > 7/2$ for $p_2 = 3 - \epsilon$, Check!].

Eg: $p_2 = 2.5$, $p_1^* = 3$, $q_1^* = 0$.

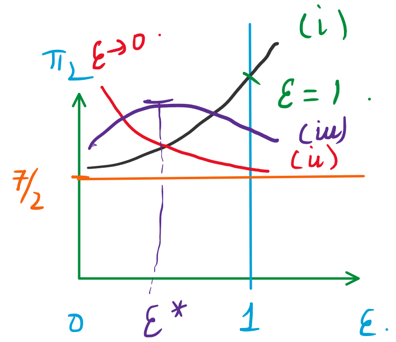
$$q_2 = 10 - p_2 = 10 - 2.5 = 7.5$$

$$\pi_2(p_1^* = 3, p_2 = 2.5) = 2.5 \times 7.5 - 2 \times 7.5$$

$$= 7.5(2.5-2)$$
$$= 7.5(0.5)$$
$$= \frac{15}{2} \cdot \frac{1}{2} = \frac{15}{4} = 3.75$$

$P_2 = 2.5$ indicates that firm will undercut.

$\therefore$ Find how much will Firm II undercut below 3.



$P_2 = 3 - \epsilon, \quad \epsilon > 0, \quad P_1^* = 3.$

$q_2 = 10 - P_2 = 10 - 3 + \epsilon = (7+\epsilon), \quad q_1^* = 0.$

$$\pi_2 = (3-\epsilon)(7+\epsilon) - 2(7+\epsilon)$$
$$= (7+\epsilon)(3-\epsilon-2) = (7+\epsilon)(1-\epsilon)$$

$\therefore$ To find the range of ϵ: $\boxed{(7+\epsilon)(1-\epsilon) > 7/2}$.

$\frac{d\pi_2}{d\epsilon} = -6 - 2\epsilon < 0$ as $\epsilon > 0$. [case (ii) on graph]

$\therefore \quad P_2^* = 3 - \epsilon, \quad \epsilon \to 0.$

$\boxed{P_1^* = 3, \quad P_2^* = 3 - \epsilon, \ \epsilon \to 0}$