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Poisson and Binomial Distribution

Q The probability that an individual will suffer a bad reaction from a particular injection is p = 0.001. Determine the probability that out of n = 2000 individuals more than 2 individuals will suffer a bad reaction.

[as $n \to \infty$ (very large) and $p \to 0$
then $np \to \lambda$ (this is binomial approximation of poison distribn)]

BD $x = 0 \cdots n$
$n \to \infty$ (large)
$p \to 0$
BD → PD
$np \to \lambda$

Soln: X denotes the number of individuals out of 2000 individuals, who suffer a bad reaction

X follows binomial dist with $n = 2000$ and $p = 0.001$.

Since n is sufficiently large and p is quite small

but $np = \lambda = 2$ moderate size.

∴ variable x may follow approximately poisson distribn with parameter $\lambda = 2$.

$$P(X = x) = \frac{e^{-2}\, 2^{x}}{x!}, \quad x = 0, 1, 2, \cdots, \infty$$

We require $P(X > 2) = 1 - P(X \leq 2)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$+ P(X=2)]$

$$= 1 - \sum_{x=0}^{2} e^{-\lambda} \frac{\lambda^x}{x!}$$

$$= 1 - \left\{ e^{-2}\frac{2^0}{0!} + e^{-2}\frac{2^1}{1!} + e^{-2}\frac{2^2}{2!} \right\}$$

$$= 1 - e^{-2}[1+2+2]$$

$$= 1 - 5e^{-2} \text{ (ans)}$$

Q2 A car hire firm has 2 cars which it hires out day by day. The number of demands for a car on each day follows Poisson Distribution with mean 1.5. Calculate the proportion of days on which

$\lambda = 1.5$

(i) neither car is used.

(ii) some demand is refused (given $e^{-1.5} = 0.2231$)

Soln: X denotes no. of demands for a car in a day. Here mean, $\lambda = 1.5$

Then $P(X=x) = e^{-\lambda} \frac{\lambda^x}{x!}$

$$= e^{-1.5} \frac{(1.5)^x}{x!} \quad x = 0, 1, 2 \ldots \infty$$

(i) $P(X=0) = e^{-1.5}(1.5)^0/0! = e^{-1.5} = 0.2231$ (ans).

(ii) $P(X>2) = 1 - P(X \le 2)$

$$= 1 - \sum_{x=0}^{2} e^{-1.5} \frac{(1.5)^x}{x!}$$

$$= 1 - \left[e^{-1.5} \frac{1.5^0}{0!} + e^{-1.5} \frac{1.5^1}{1!} + e^{-1.5} \frac{1.5^2}{2!} \right]$$

$$= 1 - e^{-1.5} \left[1 + 1.5 + \frac{(1.5)^2}{2!} \right]$$

$$= 1 - 0.2231 \times 3.625$$

$$= 0.1913 \text{ (approx).}$$

(3) If X and Y are independent Poisson variables such that

$X \sim P(\lambda_1)$

$Y \sim P(\lambda_2)$

$$P(X=2) = P(X=3)$$

and $P(Y=4) = P(Y=5)$. Find the standard deviation of $2X-Y$.

Let λ_1 and λ_2 be the parameters of distribution X and Y respectively.

Given $P(X=3) = P(X=2)$

$$\Rightarrow e^{-\lambda_1} \frac{\lambda_1^3}{3!} = e^{-\lambda_1} \frac{\lambda_1^2}{2!}$$

$$\Rightarrow \lambda_1 = \frac{3!}{2!} = 3$$

$\therefore \boxed{\lambda_1 = 3}$

Again $P(Y=4) = P(Y=5)$

$$\Rightarrow e^{-\lambda_2} \frac{\lambda_2^4}{4!} = e^{-\lambda_2} \frac{\lambda_2^5}{5!}$$

$$\Rightarrow \lambda_2 = \frac{5!}{4!} = 5 \quad \therefore \lambda_2 = 5$$

$Var(ax+y)$

$= a^2 V(x) + V(y) + 2 cov(x,y)$

x and y indep

$cov(x,y) = 0$

$Var(ax+y) = a^2 V(x) + V(y)$.

if $\lambda_1 = 3 \Rightarrow E(x) = V(x) = 3$

and $\lambda_2 = 5 \Rightarrow E(Y) = V(Y) = 5$

$Var(2x-Y) = 4\,var(x) + var(Y) - 2 Cov(x,Y)$

Since x and Y are independent variable

$\therefore Cov(X,Y) = 0$

$\therefore Var(2x-Y) = 4 var(x) + var(Y)$

$= 4\lambda_1 + \lambda_2$

$= 4 \times 3 + 5 = 12 + 5 = 17$

$\therefore SD$ of $(2x-Y) = \sqrt{Var(2x-Y)} = \sqrt{17}$ (ans)

Q4 A person takes a step forward ~~with probability.~~ 0.25 and backward with probability 0.75. What is the probability that at the end of 7 steps he will be one step away from starting point?

from starting point!

5

$n = 7$ $\quad x \to$ no. of forward step in 7 step

$X \sim$ binomial with $n = 7$, $p = 0.25$

$$P(X = x) = {}^7C_x \, p^x \, q^{7-x} \quad x = 0, \ldots, 7$$

We required $P(X = 3 \text{ or } 4)$

$$= P(X=3) + P(X=4)$$

$$= {}^7C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^4 + {}^7C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^3$$

$$= {}^7C_3 \frac{3^4}{4^7} + {}^7C_4 \frac{3^3}{(4)^7}$$

$$= \frac{3^3}{4^7}\left[3 \times \frac{7!}{3!\,4!} + \frac{7!}{3!\,4!}\right]$$

$$= \frac{3^3}{4^7}\left[\frac{3 \times 7 \times 6 \times 5}{3 \times 2} + \frac{7 \times 6 \times 5}{3 \times 2}\right]$$

$$= \frac{3^3}{4^7}\left[105 + 35\right]$$

$$= \frac{3^3}{4^7} \times 140 = \frac{3^3 \times 35}{4^6}$$

945

$$= \frac{945}{4096} \text{ (ans)}$$

find the variance
① of a symmetric binomial distribution with mean 5.

② If a random variable X follows binomial distribn with mean 2 and $E(X^2) = 28/5$, find $P(X \neq 0)$.