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Elasticity of Demand

1. Own price elasticity

$$e_p = \boxed{\frac{\partial Q}{\partial P}} \times \frac{P_0}{Q_0}$$

2. Cross price elasticity

$$\boxed{e_c^{x,y} = \frac{\partial Q^x}{\partial P_y} \times \frac{P_y^0}{Q_0^x}}$$

3. Income elasticity of demand

$$\boxed{e_m^x = \frac{\partial Q}{\partial M} \times \frac{M_0}{Q_0^x}}$$

Q. demand for: $Q_b = 4850 - 5P_b + 1.5P_p + 0.1Y$

$Y = 10\,000$ $\quad P_b = 200$ $\quad P_p = 100.$

income elasticity $e = \frac{\partial Q}{\partial Y} \cdot \frac{Y}{Q}$

Taking partial derivative of Q_b w.r.t Y

$$\frac{\partial Q_b}{\partial Y} = 0.1$$

when $Y = 10,000$

$P_b = 200$

$P_p = 100$

$Q_b = 4850 - 5 \times 200 + 1.5 \times 100 + 0.1 \times 10000$

$= 4850 - 1000 + 150 + 1000$

$Q_b = 5000$

$$\therefore e_y = \frac{\partial Q_b}{\partial Y} \times \frac{Y}{Q_b}$$

$= 0.1 \times \frac{10000}{5000} = 0.2$ (ans)

$e_y = 0.2 < 1$ (inelastic demand)

$$e^{b,p} = \frac{\partial Q_b}{} \times \frac{P_p}{}$$

Cross price $e_c^{b,p} = \frac{\partial Q_b}{\partial P_p} \times \frac{P_p}{Q_b}$

Taking partial derivative of Q_b w.r.t P_p,

$\frac{\partial Q_b}{\partial P_p} = 1.5$ and $P_p = 100$ $Q_b = 5000$

$$\therefore e_c^{b,p} = 1.5 \times \frac{1\cancel{00}}{50\cancel{00}}$$

$$= \frac{\cancel{15}\,3}{\cancel{500}\,100} = 0.03 > 0$$

Beef and Pork are substitute goods.

1. $e_c > 0 \Rightarrow$ substitute
2. $e_c < 0 \Rightarrow$ complimentary
3. $e_c = 0 \Rightarrow$ neutral goods.

own price elasticity of demand.

$$e_p = \frac{\partial Q_b}{\partial P_b} \times \frac{P_b}{Q_b}$$

Here, $\frac{\partial Q_b}{\partial P_b} = -5$, $P_b = 200$, $Q_b = 5000$

$$e_p = -5 \times \frac{20\cancel{0}}{500\cancel{0}}$$

$$= -\frac{5}{50} = -\frac{5}{10} = -0.5$$

$|e_p| = 0.5 < 1$

demand)

(inelastic demand).

Cost formulas.

$TC = \text{cost}$

$AC = \frac{TC}{Q}$

$MC = \frac{\partial TC}{\partial Q}$

Revenue formulas

$TR = P \times Q$

$AR = \frac{TR}{Q} = \frac{P \times \cancel{Q}}{\cancel{Q}} = P$

$MR = \frac{\partial TR}{\partial Q}$

Product formulas

$TP_L = Q$ or $TP_K = Q$

$AP_L = \frac{Q}{L}$ & $AP_K = \frac{Q}{K}$

$MP_L = \frac{\partial Q}{\partial L}$ & $MP_K = \frac{\partial Q}{\partial K}$

$$\boxed{TC = x^2 - 0.5xy + y^2}$$

Find the MC of x and MC of y.

$MC_x = \frac{\partial TC}{\partial x}$ and $MC_y = \frac{\partial TC}{\partial y}$

$= 2x - 0.5y$

$= -0.5x + 2y$

Revise: minimisation

F.O.C $f_1 = 0$ $f_2 = 0$

S.O.C $\rightarrow f_{11} > 0$ $f_{22} > 0$

$\rightarrow f_{11} \cdot f_{22} > f_{12}^2$

Maximisation

F.O.C $f_1 = 0$ $f_2 = 0$

S.O.C $f_{11} < 0$ $f_{22} < 0$

$f_{11} \cdot f_{22} > f_{12}^2$

$f(x,y) \quad f_{11} \cdot f_{22} > f_{12}^2$

Q. given profit function of a firm

$$\Pi = 64x - 2x^2 + 4xy - 4y^2 + 32y - 14$$

F.O.C $\quad \Pi_x = \frac{\partial \Pi}{\partial x} = \boxed{64 - 4x + 4y} = 0$

or, $\quad 4x - 4y = 64$

$x - y = 16 \quad$ — (1)

$\Pi_y = \frac{\partial \Pi}{\partial y} = 4x - 8y + 32 = 0$

$4x - 8y = -32$

$x - 2y = -8 \quad$ — (2)

Solving (1) and (2)

$$\begin{array}{r} \cancel{x} - y = 16 \\ \cancel{x} - 2y = -8 \\ (-) \quad (+) \quad (+) \\ \hline y = 24 \end{array}$$

$\therefore x = 16 + y = 16 + 24 = 40$

$\therefore x = 16$ and $y = 40$ units

S.O.C $\quad \Pi_{xx} = \frac{\partial^2 \Pi}{\partial x^2}$

S.O.C

$$\Pi_{xx} = \frac{\partial^2 \Pi}{\partial x^2} = -4 < 0$$

$$\Pi_{yy} = \frac{\partial^2 \Pi}{\partial y^2} = -8 < 0$$

$$\Pi_{xy} = \frac{\partial^2 \Pi}{\partial x \partial y} = 4$$

$$\Pi_{xx} \, \Pi_{yy} = -4 \times -8 = 32$$

$$\Pi_{xy}^2 = 16$$

$\because$ $32 > 16$

$$\Pi_{xx} \cdot \Pi_{yy} > \Pi_{xy}^2$$

$\therefore$ at $x = 24$ and $y = 40$ units

Π is maximised.

$f(x, y) = 0$

$\cancel{4}$ $\boxed{2x = y}$ ✓

$\cancel{4}$ $\boxed{2x - y = 0}$ ✓

$$y = x^2 + xy + y^3$$

explicit fn $x = y$

Implicit fn $\boxed{x - y = 0}$

Suppose $x^2 + xy + y^3 = 0$ (implicit fn)

can you arrange this function with 'x' on LHS and 'y' on R.H.S?

This implicit fn cannot be changed into explicit fn

Suppose you are give an implicit fn

$$x^2 - x + y = 0$$

then you can write $\boxed{x^2 - x = y}$

explicit fn

Implicit function rule:

Implicit function rule:

$$z = f(x, y)$$

then, $\boxed{\dfrac{dy}{dx} = -\dfrac{f_x}{f_y}}$

Ex: $x^2 + 6x - 13 - y = 0$ implicit

what is dy/dx?

$$\frac{dy}{dx} = -\frac{f_x}{f_y}$$

$f_x = 2x + 6$ and $f_y = -1$

$$\therefore \boxed{\frac{dy}{dx} = -\frac{2x+6}{-1} = 2x+6}$$

(using implicit fn rule)

Inverse rule: $\dfrac{dy}{dx} = \dfrac{1}{dx/dy}$

or, $dx/dy = \dfrac{1}{dy/dx}$

what is $\dfrac{dx}{dy} = \dfrac{1}{dy/dx} = \dfrac{1}{2x+6}$ (ans)

What is $\frac{dx}{dy} = dy/dx = \frac{1}{2x+6}$ (ans)