30 December 2023 17:02

Partial derivatives, Implicit functions, Jacobian.

Implicit functions

$x = f(t)$ $\qquad$ $y = g(t)$

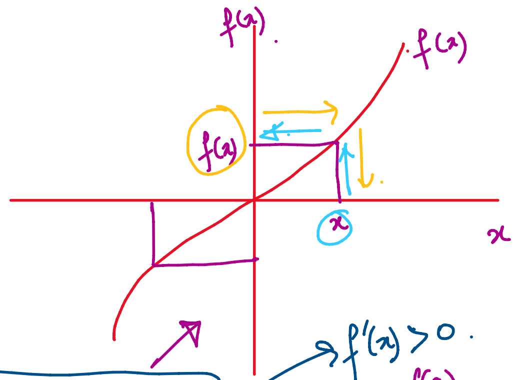
$\downarrow \qquad\qquad \downarrow$

$t = f^{-1}(x)$ $\qquad$ $t = g^{-1}(y)$

$f^{-1}(x) = g^{-1}(y)$ [f and g are both invertible]

$x = at^2$ $\qquad$ $y = \sin t$.

Condition for invertible functions.



$f(x)$ has to be a bijective function

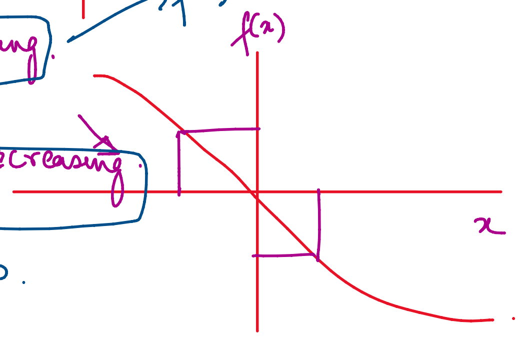
$\downarrow$

one-one and onto

$\downarrow$

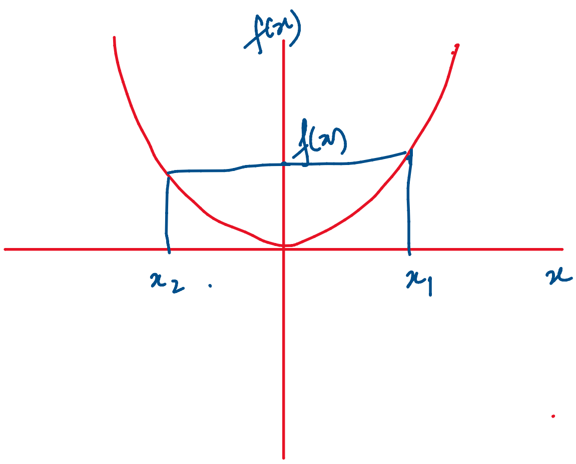
for every value of x there will be an unique value of $f(x)$ and vice versa

$\boxed{\text{monotonously increasing}}$ $\rightarrow f'(x) > 0$.



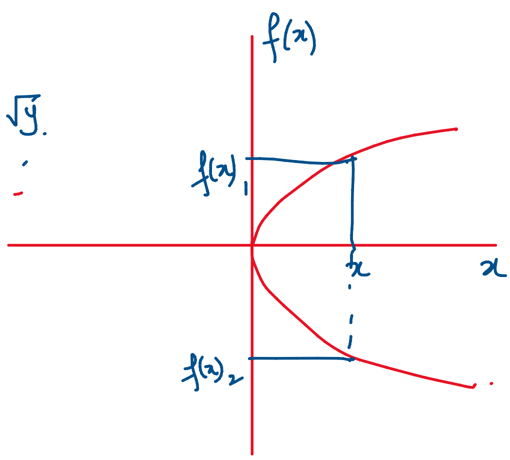
$\boxed{\text{monotonously. decreasing}}$

$f'(x) < 0$.



$y = x^2$.

$x = \pm\sqrt{y}$.



$y^2 = x$.

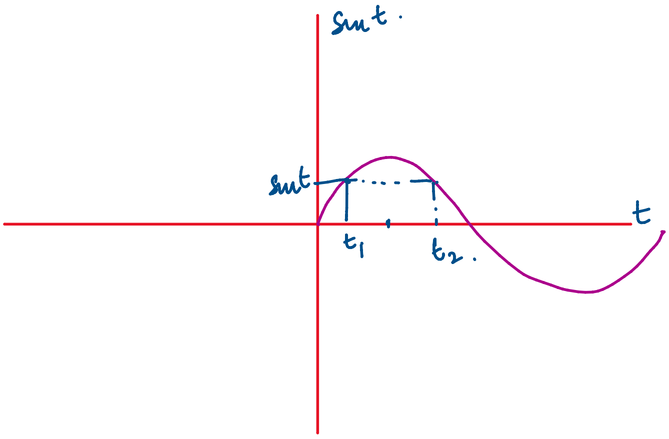
$y = \pm\sqrt{x}$.

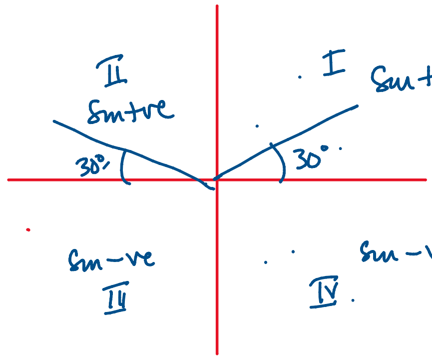
$x = t^2$ $\qquad$ $y = \sin t$. $\qquad$ $t = \sin^{-1}(\frac{1}{2})$ $\qquad$ $\sin t = \frac{1}{2}$.

$t = 30^\circ, 150^\circ$

$\sin t$.

sin t = $\frac{1}{2}$.





$t = 30°, 150°$

$x = t^2$ $\qquad$ $y = \sin t$ $\qquad$ $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t}{2t}$.

$\frac{dx}{dt} = 2t$ $\qquad$ $\frac{dy}{dt} = \cos t$.

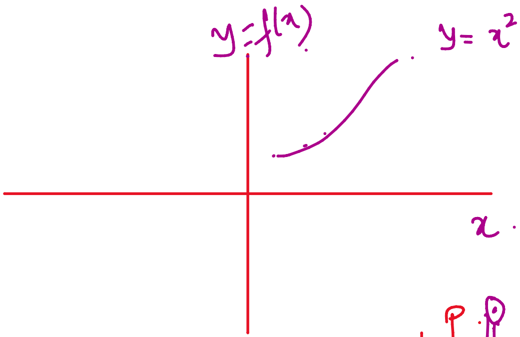
$$\boxed{u = f(x,y) \qquad V = g(x,y)}$$

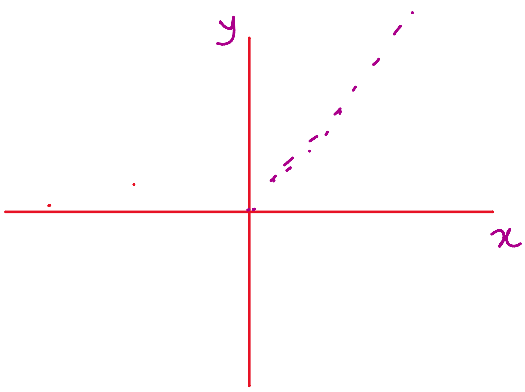
Partial derivatives $\Rightarrow$ differentiate the function wrt one of the variables keeping the other one constant.

$u = x^2 + 3xy + y^2$.

$\frac{\partial u}{\partial x} = 2x + 3y$ $\qquad$ $\frac{\partial u}{\partial y} = 3x + 2y$.

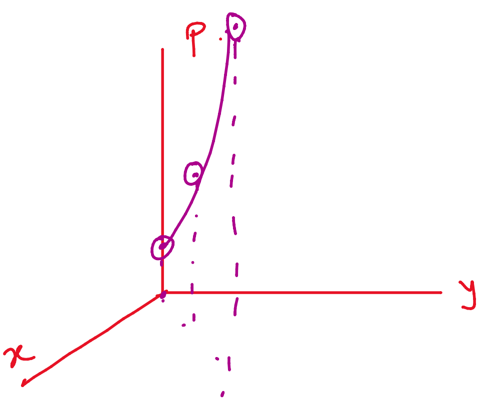
$y = x^2 + x$





P(x,y)

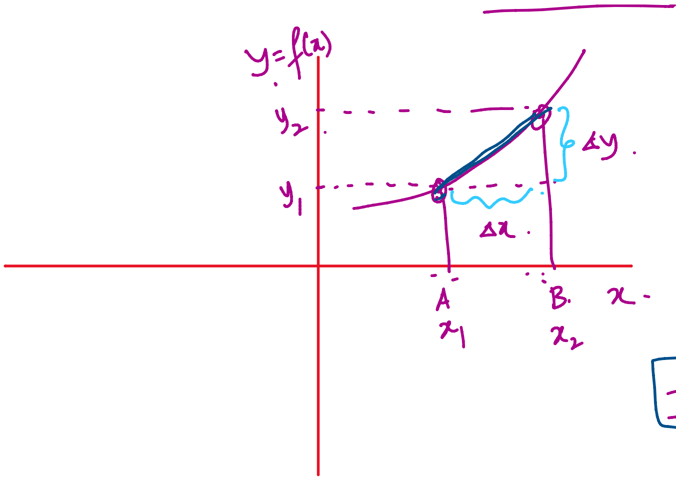
x	y	P(x,y)
0	0	1
1	1.	3.
2	2.	10



Derivatives

$\boxed{\text{Rate of change of y wrt x}}$

= Change of y $\quad \Delta y = y_2 - y_1$



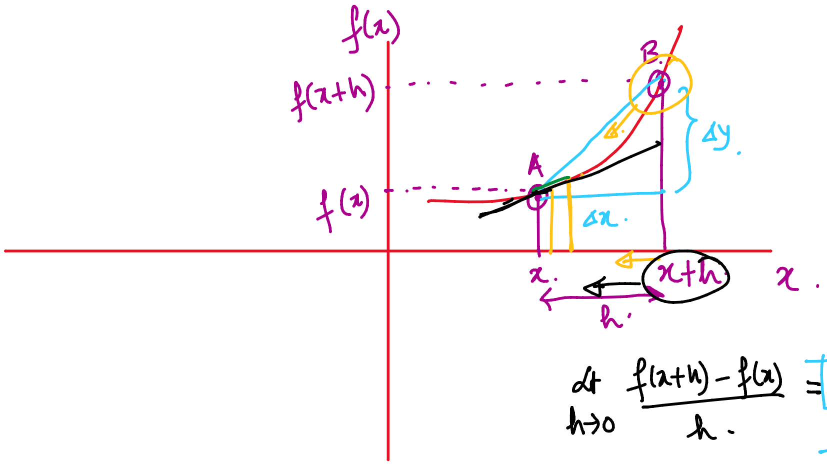
$$\boxed{\text{Rate of change of } y \text{ wrt } x}$$

$$= \frac{\text{change of } y}{\text{change of } x} = \boxed{\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}}$$

$\downarrow$

slope of the line.

$\boxed{\text{Instantaneous}}$ rate of change of y wrt x at the pt A. = ?



$$\text{rate of change} = \frac{\Delta y}{\Delta x}$$

$$= \frac{f(x+h) - f(x)}{(x+h) - x}$$

$$= \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \boxed{\text{slope of the tangent at A}}$$

$\Downarrow$

Instantaneous rate of change of $f(x)$ wrt x at A.

$\Downarrow$

$\boxed{\text{derivative of } f(x) \text{ wrt } x}$.

$$\frac{d}{dx} f(x) = f'(x)$$

$$f'(x) = \frac{d}{dx} f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$f(x) = x^2 \qquad f(x+h) = (x+h)^2 = x^2 + 2xh + h^2.$$

$$f(x+h) - f(x) = (\cancel{x^2} + 2hx + h^2) - \cancel{x^2} = 2hx + h^2$$

$$\frac{f(x+h) - f(x)}{h} = \frac{2hx + h^2}{h} = \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x + h.$$

$$\frac{d}{dx}(x^2) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} (2x+h) = 2x$$

$$\frac{d}{dx}(x^2) = 2x.$$

$$f(x) = x^n \qquad f(x+h) = (x+h)^n.$$

$f(x) = x^n \qquad f(x+h) = (x+h)^n.$

$$(x+h)^n = x^n + n.x^{n-1}h + \frac{n(n-1)}{2!}x^{n-2}h^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}h^3 + \cdots\cdots + h^n.$$

$$(x+h)^n - x^n = nx^{n-1}h + \frac{n(n-1)}{2!}x^{n-2}h^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}h^3 + \cdots\cdots + h^n.$$

$$\frac{(x+h)^n - x^n}{h} = nx^{n-1} + \frac{n(n-1)}{2!}x^{n-2}h + \frac{n(n-1)(n-2)}{3!}.x^{n-3}.h^2 + \cdots\cdots + h^{n-1}$$

$$\frac{d}{dx}(x^n) = \lim_{h\to 0}\frac{(x+h)^n - x^n}{h} = nx^{n-1} \Rightarrow \boxed{\frac{d}{dx}(x^n) = nx^{n-1}} \quad (n = \text{constant})$$

Next class.

$$\boxed{\frac{d}{dx}(\sin x) \quad \frac{d}{dx}(\cos x) \quad \frac{d}{dx}(e^x) \quad \frac{d}{dx}(\log_e x) \quad \frac{d}{dx}(a^x)}$$

$\frac{1}{2} - 1 = \frac{-1}{2}$

$x^{1/2} = \sqrt{x}.$

$\frac{d}{dx}x^5 = 5x^4$

$\frac{d}{dx}x^{1/2} = \frac{1}{2}x^{-1/2}$

$\frac{d}{dx}\sqrt{x} = \frac{1}{2}.\frac{1}{\sqrt{x}}. = \frac{1}{2\sqrt{x}}.$

$x^{-1/2} = \frac{1}{\sqrt{x}}.$