PROBABILITY 3 (14/12/2023)

Thursday, December 14, 2023 6:42 PM

Probability Problems

Non-numerical type..

Stat 6.30 (1)

54. *Three players A, B and C agree to play a series of games observing the following rules : two players participate in each game, while the third is idle, and the game is*

9062395123

to be won by one of them. The loser in each game quits and his place in the next game is taken by the player who was idle. The player who succeeds in winning over both of his opponents without interruption, wins the whole series of games. Supposing the probabilities for each player to win a single game is $\frac{1}{2}$, and that the first game is played by A and B, find the probability for A, B and C respectively to win the whole series if the number of games is unlimited.

$A_i \quad B_i \quad C_i$ ith

$P(A_i) = P(B_i) = P(C_i) = \frac{1}{2}$

A Can win the entire set

(a) A win 1st

(b) ~~B win 1st~~

$A_1 A_2$
(AB) (AC)

$A_1 C_2 B_3 A_4 A_5$
(AB) (AC) (CB) (BA) (AC)

$A_1 C_2 B_3 A_4 C_5 B_6 A_7 A_8$
AB AC BC AB AC BC AB AC

& So On

$P(\text{A's win}) = P(\text{winning in (a)}) + P(\text{win in (b)})$

$= \left\{ \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^8 + \dots \right\} + \left\{ \frac{1}{2}^4 + \frac{1}{2}^7 + \frac{1}{2}^{10} + \dots \right\}$

$= \frac{(1/2)^2}{1-(1/2)^3} + \frac{(1/2)^4}{1-(1/2)^3} = 5/14$

$\boxed{P(A), P(B) > P(C)}$

Substitute

A → B

A → C

S P R X

X 1/3

S/R P/R S/R P/R P/S R/S

(B) B wins first

$B_1 C_2 A_3 A_4$
AB BC AC AB

$B_1 C_2 A_3 B_4 C_5 A_6 A_7$

$B_1 C_2 A_3 B_4 C_5 A_6 B_7 C_8$

$A_9 A_{10}$

& So On

$P(B) = 5/14$

$P(C) = 1 - 5/14 - 5/14 = 2/7$

2. Of three independent events, the chance that the first only should happen is a, the chance of the second only is b and the chance of the third only is c. Show that the independent chances of the three events are respectively : $\frac{a}{a+x}$, $\frac{b}{b+x}$, $\frac{c}{c+x}$, where x is the root of the equation : $\boxed{(a+x)(b+x)(c+x) = x^2}$.

where x is the root of the equation : $\boxed{(a + x)(b + x)(c + x) = x^2.}$

$P(\overline{E_1}\,\overline{E_2}\,\overline{E_3}) = a$ $P(\overline{E_1})P(\overline{E_2})P(\overline{E_3}) = d$ ... (4)

$P(c) = 1 - 3/4$ ... $= 247$

$P(E_1)P(E_2)P(E_3)x^2 = abc$ ... (4)

here $x = P(\overline{E_1})P(\overline{E_2})P(\overline{E_3})$ ... (5)

$1 \div 5$

$$\frac{P(E_1)}{P(\overline{E_1})} = \frac{a}{x}$$

or, $x\,P(E_1) = a[1 - P(E_1)]$

or, $(a+x)P(E_1) = a$

$P(E_1) = \left(\frac{a}{a+x}\right)$ -- (6)

Smly (2) ÷ (6) & (3) ÷ (5)

$P(E_2) = \frac{b}{b+x}$

$P(E_3) = \frac{c}{c+x}$ -- (7)

(7) ÷ (4)

$$x^2 = \frac{abc}{P(E_1)P(E_2)P(E_3)} = \frac{abc}{\frac{a}{a+x}\cdot\frac{b}{b+x}\cdot\frac{c}{c+x}}$$

$= (a+x)(b+x)(c+x)$

Proved

151   3   100   25   9062395123

60. *A thief has a bunch of n keys, exactly one of which fits a lock. If the thief tries to open the lock by trying the keys at random, what is the probability that he requires exactly k attempts, if he rejects the keys already tried ? Find the same probability if he does not reject the keys already tried.*

Case : 1   WOR

$P(\text{1st Success}) = 1/n$

$P(\text{2nd}) = 1/(n-1)$

$P(i\text{th}) = \frac{1}{n-(i-1)}$

$P(2) = $ Prob of 1st success in 2nd trial $= \left(1 - \frac{1}{n}\right)\cdot\frac{1}{(n-1)} = \frac{1}{n}$

$P(3) = $ Prob of 1st Succ in 3rd Attempt $= \left(1 - \frac{1}{n}\right)\left(1 - \frac{1}{n-1}\right)\left(\frac{1}{n-2}\right) = 1/n$

$P(K) = $ Prob of 1st Success in Kth attempt $= 1/n$

Case II

Keys not eliminated

Case II

~~Un~~ ~~KDR~~ not eliminated

$p = 1/n$

$q = 1 - 1/n$

$P(k) = P[\text{failure in 1st } (k-1) \cap \text{success in } k\text{th}]$

$= q^{k-1} p = \left(1 - \frac{1}{n}\right)^{k-1} \frac{1}{n}$

$= \frac{(n-1)^{k-1}}{n^k}$

Bayes theorem

Q.4

~~5~~. *The chances of X, Y, Z becoming managers of a certain company are 4 : 2 : 3. The probabilities that bonus scheme will be introduced if X, Y, Z become managers, are 0·3, 0·5 and 0·8 respectively. If the bonus scheme has been introduced, what is the probability that X is appointed as the manager.*

$P(X) = 4/9 \quad P(Y) = 2/9 \quad P(Z) = 3/9$

$P(A/X) = 0.3 \quad P(A/Y) = 0.5 \quad P(A/Z) = 0.8$

$P(X)P(A/X) = \frac{1.2}{9} \quad P(Y)P(A/Y) = 2/9 \cdot 0.5 = 1/9$

$P(Z)P(A/Z) = 2.4/9$

$P(A) = \sum P(X)\,P(A/X) = \frac{1.2 + 1 + 2.4}{9} = \frac{4.6}{9}$

$P(X/A) = \frac{P(X)P(A/X)}{P(A)} = \frac{1.2/9}{4.6/9} = \frac{12}{46} = 6/23$

Q.5

~~6~~. *A bag contains (n + 1) coins. It is known that one of these coins has a head on both the sides whereas the remaining coins are fair. One of these coins is selected at random and is tossed. If the probability that the toss results in a head is (5/9), find the value of n.*

A → Head

E_1 A fair coin $\quad E_2$ two heads coin

$P(A/E_1) = 1/2$

$P(E_2) = \frac{1}{\ldots}$

A fair coin E_1 ... E_2 two headed coin

$P(E_1) = \frac{n}{n+1}$ $P(E_2) = \frac{1}{n+1}$

$P(A|E_1) = 1/2$

$P(A|E_2) = 1$

$P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2) = 5/9$

$\frac{1}{2} \cdot \frac{n}{n+1} + \frac{1}{n+1} \cdot 1 = 5/9$

$9(n+2) = 10(n+1)$

$\boxed{n = 8}$

$E_1 \to A, B$ agree (True) $= 2/5$

$E_2 \to A, B \to n$ (False) $= 3/5$

$E \to A, B$ agree

Q.7

2. A and B are two independent witnesses (i.e., there is no collusion between them) in a case. The probability that A will speak the truth is p_1 and for B it is p_2. A and B agree in a certain statement. Show that the probability that this statement is true is :

$p_1p_2/[1 - p_1 - p_2 + 2p_1p_2]$

$E \to (E \cap E_1) \cup (E \cap E_2)$

$= p_1 \cdot p_2 + (1-p_1)(1-p_2) = 1 - p_1 - p_2 + 2p_1p_2$

$P(E_1|E) = \frac{P(E \cap E_1)}{P(E)} = \frac{p_1p_2}{1 - p_1 - p_2 + 2p_1p_2}$

3. A and B are two very weak students of Statistics and their chances of solving a problem correctly are 1/8 and 1/12 respectively. If the probability of their making a common mistake is 1/1001 and they obtain the same answer, find the chance that their answer is correct.

$E_1 \rightarrow$ A, B Same Ans ✓

$E_2 \rightarrow$ Same ✗

$E \rightarrow$ Same ans

$E = (E \cap E_1) \cup (E \cap E_2)$

$= \frac{1}{8} \times \frac{1}{12} + \left(1 - \frac{1}{8}\right)\left(1 - \frac{1}{12}\right) \cdot \frac{1}{1001}$

$= \frac{1}{96} + \frac{7}{8} \cdot \frac{11}{12} \cdot \frac{1}{1001}$

$= \frac{1}{96}\left(1 + \frac{1}{13}\right) = \frac{1}{96} \times \frac{14}{13}$

$P(E_1 | E)$

$= \frac{P(E \cap E_1)}{P(E)} = \frac{1/96}{14/(96 \times 13)}$

$= 13/14$