Integral Calculus

Summary

(i) Basics - Properties of Definite Integrals.

(ii) Leibnitz Rule - Differentiate an Integral.

(iii) Beta and Gamma Functions

(iv) Double Integral

↳ Changing order of integration (when range of x, y are dependent)

↳ Triple Integrals.

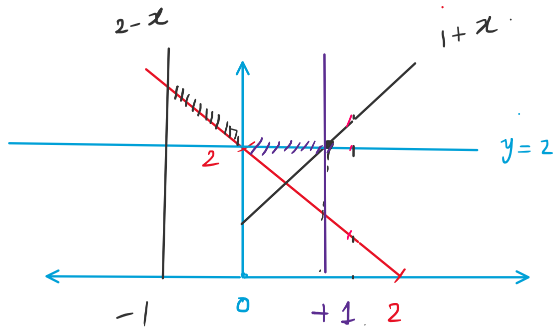
Q. Let V be the region bounded by the planes $x=0$, $x=2$, $y=0$, $z=0$ and $y+z=1$. Find $\iiint_V y\,dx\,dy\,dz$.

$$\int_0^1\int_0^{1-z}\int_0^2 y\,dx\,dy\,dz = \frac{1}{3}.$$

Q. Let $f(x) = \max\{2-x, 2, 1+x\}$. Then $\int_{-1}^{1} f(x)\,dx =$

(a) 0 (b) 2 ~~(c)~~ $9/2$ (d) None.

$$f(x) = \begin{cases} 2-x, & -1 \le x < 0 \\ 2, & 0 \le x \le 1 \\ 1+x, & x \ge 1. \end{cases}$$



$$\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$$

$$= \int_{-1}^{0}(2-x)\,dx + \int_0^1 2\,dx = 9/2 \ (c)$$

Q. If $\int_0^1 x e^{x^2} dx = k\int_0^1 e^{x^2} dx$, then: (a) $k>1$ ~~(b)~~ $0<k<1$ (c) $k=1$ (d) None

$\int_0 e \, dx$, then: (a) $k > 1$ ~~(b)~~ $0 < k < 1$

(c) $k = 1$ (d) None.

$\int_0^1 x e^{x^2} dx$ let $x^2 = t$

$\frac{1}{2}\int_0^1 e^t dt = k \cdot \int_0^1 e^{x^2} dx$

$\int_0^1 e^{x^2} dx$ $x^2 = t \Rightarrow x = \sqrt{t}$

$2x\,dx = dt$

$\frac{1}{2}\int_0^1 \frac{1}{\sqrt{t}} \cdot e^t dt$ $2\sqrt{t}\,dx = dt$

$dx = \frac{dt}{2\sqrt{t}}$

$\frac{1}{2}\int_0^1 \underset{\mathrm{I}}{t^{-1/2}} \underset{\mathrm{II}}{e^t} dt$

$t^{-1/2}\int e^t dt - \int -\frac{1}{2} t^{-3/2} e^t dt$.

For the integration:

$0 < x < 1$.

$0 < x e^{x^2} < e^{x^2}$

$\int_0^1 0\,dx < \int_0^1 x e^{x^2} dx < \int_0^1 e^{x^2} dx$

$0 < \int_0^1 x e^{x^2} dx < \int_0^1 e^{x^2} dx$.

$\quad\quad \|$

$0 < k\int_0^1 e^{x^2} dx < \int_0^1 e^{x^2} dx$

Divide $\int_0^1 e^{x^2} dx$: $\boxed{0 < k < 1}$ (b).

Q. If $a_n = \int_0^{\pi} \frac{\sin\{(2n-1)x\}}{\sin x} dx$, then $a_1, a_2, a_3, \ldots$ are in

(a) AP & HP (b) AP, GP but not HP (c) GP, HP (d) AP, GP, HP

Note: Check for AP: Find c.d (d) ✓

Check for GP: Find c.r (r) ✓

Check for HP: Find c.d of reciprocals $\left(\frac{1}{d}\right)$

Given a seq: $\{a_1, a_2, a_3, \ldots, a_n, a_{n+1}, \ldots\}$

Consider $a_n = \int^{\pi} \frac{\sin(2n-1)x}{\sin x} dx$ $a_{n+1} = \int^{\pi} \frac{\sin(2n+1)x}{} dx$

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$$a_{n+1} - a_n = \int_0^{\pi} \frac{\sin(2n+1)x - \sin(2n-1)x}{\sin x}\,dx$$

[Note: $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)$]

$$a_{n+1} - a_n = 2\int_0^{\pi} \cos 2nx\,dx = 2\left[\frac{\sin 2nx}{2n}\right]_0^{\pi}$$

$$= \frac{1}{n}\left[\sin 2\pi n - 0\right] = 0$$

$a_{n+1} - a_n = 0 \Rightarrow \boxed{a_{n+1} = a_n \ \forall n}$

Q. Let $f(x)$ be an integrable odd fn in $[-5,5]$ s.t $f(x+10) = f(x)$

then: $\int_x^{x+10} f(t)\,dt = ?$ $\qquad$ (a) 0 $\qquad$ (b) $2\int_x^5 f(t)\,dt$

(c) > 0 $\qquad$ (d) None.

Let $y = \int_x^{x+10} f(t)\,dt$

Applying Leibnitz Rule: $\frac{dy}{dx} = f(x+10) - f(x) = 0$.

$\frac{dy}{dx} = 0 \Rightarrow y$ is a constant

y is independent of x.

(i)

Given: $y = \int_x^{x+10} f(t)\,dt$

Put $x = -5$: $\quad y = \int_{-5}^{5} f(t)\,dt = 0$., $x \in [-5,5]$ ---- (ii)

$x+10$

$$\int_{-5} \ldots \quad x \in [-5,5] \text{ ---- (ii)}$$

$$\therefore \int_{x}^{x+10} f(x)\,dx = 0 \quad \forall x \in [-5,5]$$