

If  $f(x) = x^2 - 7x$  and  $g(x) = x + 3$ , then the minimum value of  $f(g(x)) - 3x$  is

$$f(x) = x^2 - 7x$$

$$f(g(x)) = [g(x)]^2 - 7[g(x)]$$

$$= [x+3]^2 - 7[x+3]$$

$$= x^2 + 6x + 9 - 7x - 21$$

$$= x^2 - x - 12$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(x+3)^2 = x^2 + 6x + 9$$

$$f[g(x)] - 3x = x^2 - x - 12 - 3x$$

$$= x^2 - 4x - 12$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(x-2)^2 = x^2 - 4x + 4$$

$$x^2 - 4x = (x-2)^2 - 4$$

$$= (x-2)^2 - 4 - 12$$

$$= (x-2)^2 - 16$$

$$\geq -16$$

function

$y = x + 2$ .  $y$  is a function of  $x$ .

$$f(x) = x + 2$$

if  $x = 3$   $f(x) = 3 + 2 = 5$   
 if  $x = -1$   $f(x) = -1 + 2 = 1$

$$f(b) = b + 2$$

$$x^2 - 4x = x^2 - 2 \times 2 \times x$$

$$x^2 \geq 0$$

$$(x-2)^2 \geq 0$$

min value of  $(x-2)^2 = 0$

min value of  $f[g(x)] - 3x$  is  $-16$

If  $3x + 2|y| + y = 7$  and  $x + |x| + 3y = 1$ , then  $x + 2y$  is

Case 1  $y \geq 0, |y| = y$

$$3x + 2y + y = 7$$

$$3x + 3y = 7 \quad \text{--- (1)}$$

Case 1  $x > 0, |x| = x$

$$x + x + 3y = 1$$

$$2x + 3y = 1 \quad \text{--- (3)}$$

Case 2  $y < 0, |y| = -y$

$$3x + 2(-y) + y = 7$$

$$3x - 2y + y = 7$$

$$3x - y = 7 \quad \text{--- (2)}$$

Case 2  $x < 0, |x| = -x$

$$x + (-x) + 3y = 1$$

$$x - x + 3y = 1$$

$$3y = 1 \quad \text{--- (4)}$$

$$2x + 3y = 1 \quad \checkmark$$

$$3 \times (3x - y = 7)$$

$$9x - 3y = 21 \quad \checkmark$$

$$y = \frac{1}{3} \quad \text{--- (4)}$$

$$2x + 3y = 1$$

$$9x - 3y = 21$$

$$\hline 11x = 22 \quad \checkmark$$

$$x = 2 \quad \checkmark$$

$|y| = \text{absolute value of } y$ .

$$|2| = 2 \quad |-2| = 2 \quad |-2| = -(-2)$$

$|y| = y$  when  $y \geq 0$   
 $= -y$  when  $y < 0$ .

$$3x + 3y = 7$$

$$-(2x + 3y = 1)$$

$$\hline x = 6$$

$$3x + 3y = 7$$

$$3x + 1 = 7$$

$$3x = 7 - 1 = 6$$

$$3x = 6$$

$$x = 2 \quad \times$$

$$2x + 3y = 1$$

$$2 \times 6 + 3y = 1$$

$$12 + 3y = 1$$

$$3y = -11$$

$$y = -\frac{11}{3} \quad \times$$

$$2x + 3y = 1$$

$$2 \times 2 + 3y = 1$$

$$4 + 3y = 1$$

$$3y = -3$$

$$y = -1 \quad \checkmark$$

$$x + 2y = 2 + 2 \times (-1) = 2 - 2 = 0 \quad \text{Ans.}$$

Total.

Among 100 students  $x_1$  have birthdays in January,  $x_2$  have birthday in February, and so on. If  $x_0 = \max(x_1, x_2, \dots, x_{12})$ , then the smallest possible value of  $x_0$  is

|                | Jan   | Feb   | Mar   | Apr   | May   | Jun   | Jul   | Aug   | Sep   | Oct      | Nov      | Dec.     | Total |
|----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|----------|----------|-------|
| No of students | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ | $x_{11}$ | $x_{12}$ | 100   |
| →              | 5     | 10    | 5     | 15    | 10    | 5     | 5     | 15    | 5     | 5        | 10       | 10       | 100   |

$x_0 = 15$

|   |    |    |    |    |   |   |    |    |    |    |   |   |  |
|---|----|----|----|----|---|---|----|----|----|----|---|---|--|
| → | 10 | 10 | 10 | 10 | 5 | 5 | 10 | 10 | 10 | 10 | 5 | 5 |  |
|---|----|----|----|----|---|---|----|----|----|----|---|---|--|

$x_0 = 10$

min of  $x_0 = 10$

$\frac{100}{12} = 8 + \frac{4}{3}$

$\frac{4 \times 9}{36} + \frac{8 \times 8}{64}$

$x_0 = \max(x_1, x_2, \dots, x_{12})$

$\min(x_0) = \min[\max(x_1, x_2, \dots, x_{12})]$

$x_1, x_2, \dots, x_{12}$  has to be as close as possible.

|   |   |   |   |   |   |   |   |   |   |   |   |   |   |     |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|-----|
| 9 | 9 | 9 | 9 | 9 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|-----|

$x_0 = 9$

$\min(x_0) = 9$

The number of real-valued solutions of the equation  $2^x + 2^{-x} = 2 - (x - 2)^2$  is

How many 3-digit numbers are there, for which the product of their digits is more than 2 but less than 7?

$\underline{a=4}$   
 $\underline{bc=1}$      $\underline{4}$      $\underline{\quad}$      $\underline{\quad}$     ①  
           1    1  
 $\underline{a=5}$   
 $\underline{bc=1}$      $\underline{5}$      $\underline{\quad}$      $\underline{\quad}$     ①  
           1    1  
 $\underline{a=6}$   
 $\underline{bc=1}$      $\underline{6}$      $\underline{\quad}$      $\underline{\quad}$     ①  
           1    1

② numbers

$\underline{\quad}$      $\underline{\quad}$      $\underline{\quad}$   
 $a$      $b$      $c$   
 $\underline{1-9}$      $\underline{0-9}$      $\underline{0-9}$   
 $\underline{1}$   
       1    3  
       3    1  
       1    4  
       4    1    ⑪  
       2    2  
       1    5  
       5    1  
       1    6  
       6    1  
       2    3  
       3    2

$2 < abc < 7$

$3 \leq abc \leq 6$

$abc \rightarrow 3, 4, 5, 6$

①  $a=1$      $bc = 3, 4, 5, 6$

$\underline{a=2}$      $\underline{2}$      $\underline{\quad}$      $\underline{\quad}$   
 $\underline{bc=2,3}$     1    2  
               2    1    ④  
               1    2  
               3    1

$\underline{a=3}$      $\underline{3}$      $\underline{\quad}$      $\underline{\quad}$   
 $\underline{bc=1,2}$     1    1    ③  
               1    2  
               2    1

The number of distinct real roots of the equation  $\left(x + \frac{1}{x}\right)^2 - 3\left(x + \frac{1}{x}\right) + 2 = 0$  is

If  $a$ ,  $b$  and  $c$  are positive integers such that  $ab = 432$ ,  $bc = 96$  and  $c < 9$ , then the smallest possible value of  $a + b + c$  is

How many distinct positive integer-valued solutions exist to the equation  $(x^2 - 7x + 11)^{(x^2 - 13x + 42)} = 1$ ?