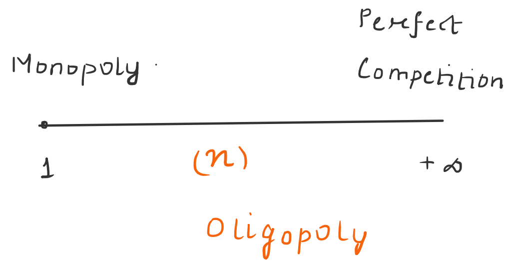
Oligopoly.



Mkt structure with "strategic interaction."
[formulating strategies to compete in the market].

Mkt structure is possible when we have a small no. of firms in the mkt.

Strategy on Quantity [Quantity Competition].
=> Cournot Model, Stakelberg Model
Strategy on Prices [Price Competition].
=> Bertrand Model.

Cournot Model:

Each firm simultaneously decides on the level of production and compete in the mkt.

Eg: Consider 2 firms. Firm I: output level q_1 and $C_1(q_1) = c_1 \cdot q_1$,
Firm II: output level q_2 and $C_2(q_2) = c_2 \cdot q_2$.
Total output: $q = q_1 + q_2$.
Let the mkt demand curve be: $P = a - bq$, $a, b > 0$.
Find the Cournot level of output of both firms.

Firm I: $\pi_1 = R_1 - C_1 = P \cdot q_1 - C_1$

$$\pi_1 = [a - b(q_1 + q_2)] \cdot q_1 - c_1 \cdot q_1$$

Firm I will choose q_1 to max π_1:

$$\frac{\partial \pi_1}{\partial q_1} = 0 \Rightarrow [a - b(q_1 + q_2)] + q_1(-b) - c_1 = 0.$$

$$\Rightarrow (a - c_1) - 2bq_1 - bq_2 = 0.$$

$$\Rightarrow \quad 2bq_1 + bq_2 = (a-c_1) \quad ----(i)$$

Firm II: $\pi_2 = R_2 - C_2 = P \cdot q_2 - C_2$

$$\pi_2 = [a - b(q_1+q_2)] \cdot q_2 - c_2 q_2$$

Firm II will choose q_2 to max π_2:

$$\frac{\partial \pi_2}{\partial q_2} = 0 \Rightarrow [a - b(q_1+q_2)] - bq_2 - c_2 = 0$$

$$\Rightarrow \quad bq_1 + 2bq_2 = (a-c_2) \quad ----(ii)$$

Using (i) & (ii) solve for q_1^* and q_2^*:

$$\boxed{2b}\,q_1 + \boxed{b}\,q_2 = (a-c_1)$$
$$\boxed{b}\,q_1 + \boxed{2b}\,q_2 = (a-c_2)$$

Cramer's Rule:-

$$q_1^* = \frac{\begin{vmatrix} (a-c_1) & b \\ (a-c_2) & 2b \end{vmatrix}}{\begin{vmatrix} 2b & b \\ b & 2b \end{vmatrix}} \qquad q_2^* = \frac{\begin{vmatrix} 2b & (a-c_1) \\ b & (a-c_2) \end{vmatrix}}{\begin{vmatrix} 2b & b \\ b & 2b \end{vmatrix}}$$

$$= \frac{2b(a-c_1) - b(a-c_2)}{3b^2} \qquad = \frac{2b(a-c_2) - b(a-c_1)}{3b^2}$$

$$= \frac{2a - 2c_1 - a + c_2}{3b} \qquad = \frac{2a - 2c_2 - a + c_1}{3b}$$

$$q_1^* = \frac{a - 2c_1 + c_2}{3b} \qquad q_2^* = \frac{a - 2c_2 + c_1}{3b}$$

$$p^* = a - b(q_1^* + q_2^*)$$

$$= a - \cancel{b}\left[\frac{2a - c_1 - c_2}{3\cancel{b}}\right] = \frac{a + c_1 + c_2}{3}$$

$$\pi_1^* = p^* q_1^* - c_1 \cdot q_1^* = (p^* - c_1) \cdot q_1^* = \left(\frac{a+c_1+c_2}{3} - c_1\right)\left(\frac{a-2c_1+c_2}{3b}\right)$$

$$= \left(\frac{a-2c_1+c_2}{3}\right)\left(\frac{a-2c_1+c_2}{3b}\right)$$

$$= \frac{(a-2c_1+c_2)^2}{9b}.$$

$$\pi_2^* = p^* q_2^* - c_2 \cdot q_2^* = (p^* - c_2) \cdot q_2^* = \left(\frac{a+c_1+c_2}{3} - c_2\right)\left(\frac{a-2c_2+c_1}{3b}\right)$$

$$= \frac{(a-2c_2+c_1)^2}{9b}.$$

Find the condition under which both firms will have identical profits in the Cournot Model.

$$\pi_1^* = \pi_2^* \Rightarrow \frac{(a-2c_1+c_2)^2}{\cancel{9b}} = \frac{(a-2c_2+c_1)^2}{\cancel{9b}}$$

$$\Rightarrow \cancel{a} - 2c_1 + c_2 = \cancel{a} - 2c_2 + c_1$$

$$\Rightarrow \boxed{c_1 = c_2}$$

Does identical MC $\Rightarrow$ identical output level? [In Cournot Model]

If $c_1 = c_2 \Rightarrow q_1^* = q_2^*$.

If $c_1 = c_2 = c > 0$

$$q_1^* = \frac{a-c}{3b}, \quad q_2^* = \frac{a-c}{3b}, \quad p^* = \frac{a+2c}{3}.$$

$$\pi_1^* = \frac{(a-c)^2}{9b} = \pi_2^*$$

Results from the Cournot Model.

Q. Suppose Firm I and Firm II collude to form a cartel. Find the output levels of both firms in the cartel and compare with the Cournot output level. [Assume: $c_1 = c_2 = c$].

Cartel: merger of firms to behave as a single unit.

So cartel, all the participating firms are concerned with maximizing the joint profit / combined profit.

Joint profit $\pi = R_1 + R_2 - C_1 - C_2$.

$$\pi = P \cdot q_1 + P \cdot q_2 - c \cdot q_1 - c \cdot q_2$$

$$\pi = P(q_1 + q_2) - c(q_1 + q_2).$$

$$\pi = [a - b(q_1 + q_2)](q_1 + q_2) - c \cdot (q_1 + q_2).$$

Firm I would choose q_1 to max π [under cartel].

HW $\frac{\partial \pi}{\partial q_1} = 0 \Rightarrow$