True Model: $Y_i = \alpha + \beta X_i + u_i$

To test: $H_0: \beta = 0$ vs $H_1: \beta \neq 0$.

$$\frac{(\hat{\beta}-\beta)^2}{\sigma^2/\Sigma(x_i-\bar{x})^2} \sim \chi^2_{(1)}$$

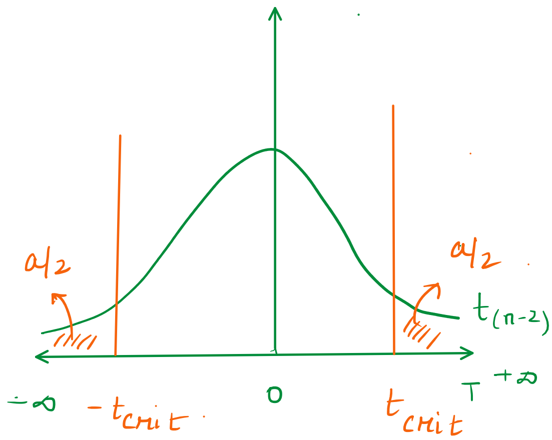
$$\frac{\Sigma e_i^2}{\sigma^2} \sim \chi^2_{(n-2)}$$

Test-statistic $F = \dfrac{(\hat{\beta}-\beta)^2}{\hat{\sigma}^2/\Sigma(x_i-\bar{x})^2} \sim F_{1,(n-2)}$.

$$\therefore \sqrt{F} = \sqrt{\frac{(\hat{\beta}-\beta)^2}{\hat{\sigma}^2/\Sigma(x_i-\bar{x})^2}} = \frac{\hat{\beta}-\beta}{\hat{\sigma}/\sqrt{\Sigma(x_i-\bar{x})^2}} = \frac{\hat{\beta}-\beta}{s.e(\hat{\beta})} \sim t_{(n-2)}$$

Test-statistic: $T = \dfrac{\hat{\beta}-\beta}{s.e(\hat{\beta})} \sim t_{(n-2)}$

Under H_0: $T_{H_0} = \dfrac{\hat{\beta}}{s.e(\hat{\beta})} \sim t_{(n-2)}$



Testing Rule: Fix a level of significance $a\%$.

We will reject H_0 at $a\%$ L.O.S if

$T_{obs} > t_{crit}$ or $T_{obs} < -t_{crit}$

$\Rightarrow |T_{obs}| > t_{crit}$.

Recall: ANOVA:

$$TSS = ESS + RSS.$$

$$\Sigma(Y_i-\bar{Y})^2 = \Sigma(\hat{Y}_i-\bar{Y})^2 + \Sigma(\underbrace{Y_i-\hat{Y}_i}_{e_i})^2 = \Sigma e_i^2$$

$$\Sigma(Y_i-\bar{Y})^2 = \Sigma(\hat{Y}_i-\bar{Y})^2 + \Sigma e_i^2$$

d.f of ESS =

d.f of RSS = .

Define d.f = Total no. of obs - Total no. of restriction /

Total no. of independent obs.

$$RSS = \sum_{i=1}^{n} e_i^2 = e_1^2 + e_2^2 + \cdots + e_n^2$$ $\Rightarrow$ n terms in the summation. (Total no. of values).

$\Rightarrow$ 2 restrictions on values of e:

$\sum e_i = 0, \quad \sum e_i X_i = 0.$

$\Rightarrow$ d.f = $(n-2)$

$$ESS = \sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2 = \sum_{i=1}^{n}\left\{\hat{\beta}(X_i - \bar{X})\right\}^2 = \hat{\beta}^2 \sum_{i=1}^{n}(X_i - \bar{X})^2.$$

Here: $\hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i$

$\Rightarrow \bar{\hat{Y}} = \hat{\alpha} + \hat{\beta}\bar{X} \Rightarrow \bar{Y} = \hat{\alpha} + \hat{\beta}\bar{X}$

$\therefore (\hat{Y}_i - \bar{Y}) = \hat{\beta}(X_i - \bar{X})$

$Y_i = \hat{Y}_i + e_i$

$\bar{Y} = \bar{\hat{Y}} + \bar{e}$, $\bar{e} = 0$.

$\therefore$ d.f of ESS = 1.

Q. True Model: $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$

Find d.f of ESS and RSS.

Estimated model: $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i}$

From ANOVA: TSS = ESS + RSS.

$$\sum(Y_i - \bar{Y})^2 = \sum(\hat{Y}_i - \bar{Y})^2 + \sum e_i^2$$

RSS: $\sum_{i=1}^{n} e_i^2 = e_1^2 + e_2^2 + \cdots + e_n^2 \Rightarrow$ 'n' terms in the summation.

$\Rightarrow$ 3 restrictions of e_i values:

$\sum e_i = 0, \ \sum e_i X_{2i} = 0, \ \sum e_i X_{3i} = 0.$

$\therefore$ d.f of RSS $= (n-3)$

$$ESS: \sum_{i=1}^{n} \underbrace{(\hat{Y}_i - \bar{Y})^2}_{} = \sum_{i=1}^{n} \left\{ \hat{\beta}_2 (X_{2i} - \bar{X}_2) + \hat{\beta}_3 (X_{3i} - \bar{X}_3) \right\}^2$$

$$\longrightarrow (\hat{Y}_1 - \bar{Y})^2 + (\hat{Y}_2 - \bar{Y})^2 + \cdots + (\hat{Y}_n - \bar{Y})^2.$$

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i}$$

$$\bar{\hat{Y}} = \hat{\beta}_1 + \hat{\beta}_2 \bar{X}_2 + \hat{\beta}_3 \bar{X}_3 \Rightarrow \bar{Y} = \hat{\beta}_1 + \hat{\beta}_2 \bar{X}_2 + \hat{\beta}_3 \bar{X}_3$$

$$\therefore \boxed{(Y_i - \bar{Y})} = \hat{\beta}_2 (X_{2i} - \bar{X}_2) + \hat{\beta}_3 (X_{3i} - \bar{X}_3)$$

$\therefore$ d.f of ESS $= 2$.

True Model: $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$

Parameters: $\beta_1, \beta_2, \beta_3$.

Explanatory variables: (X_2, X_3).

1) $H_0: \beta_1 = 0 \quad \beta_1 \neq 0$

2) $H_0: \beta_2 = 0 \quad \beta_2 \neq 0$

3) $H_0: \beta_3 = 0 \quad \beta_3 \neq 0$.

To test: $H_0: \beta_2 = \beta_3 = 0$. vs H_1: atleast one of $\beta_2 / \beta_3 \neq 0$.

d.f of ESS $= 2$. $\Rightarrow \dfrac{ESS}{\sigma^2} \sim \chi^2_{(2)}$

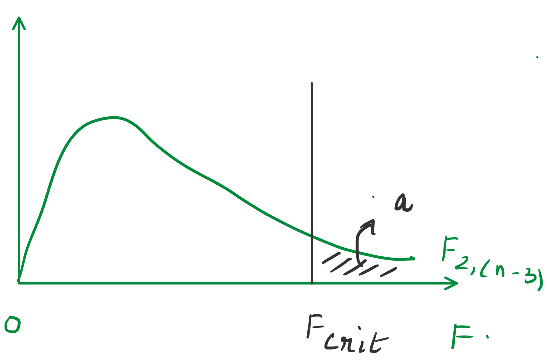
d.f of RSS $= (n-3) \Rightarrow \dfrac{RSS}{\sigma^2} \sim \chi^2_{(n-3)}$

$\rightarrow$ independent.

$$F = \frac{\frac{ESS}{\cancel{\sigma^2}} / 2}{\frac{RSS}{\cancel{\sigma^2}} / (n-3)} \sim F_{2,(n-3)}$$

$$F = \frac{ESS/2}{RSS/(n-3)} \sim F_{2,(n-3)}$$

$\Rightarrow$ Test-statistic.



Testing Rule: Fix level of significance $= a\%$.

Smaller value of F $\Rightarrow$ Accept H_0.

Smaller value of F ⇒ Accept H_0.
Larger value of F ⇒ Reject H_0.

Reject H_0 at a% l.o.s if $F_{obs} > F_{crit}$.