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Proof ① $AM \geq GM \geq HM$.

Let $x_1, x_2, x_3, \cdots, x_n$ be a set of n observations

Their AM (A), GM (G) and HM (H) are

$$A = \frac{x_1 + x_2 + \cdots + x_n}{n} \quad \text{---} \ (1)$$

$$G = (x_1 x_2 x_3 \cdots x_n)^{1/n} \quad \text{---} \ (2)$$

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}} \quad \text{---} \ (3)$$

Let us consider two observations x_1 and x_2

$$\left(\sqrt{x_1} - \sqrt{x_2}\right)^2 \geq 0$$

or, $x_1 - 2\sqrt{x_1}\sqrt{x_2} + x_2 \geq 0$

or, $\frac{x_1 + x_2}{2} \geq \sqrt{x_1 \cdot x_2}$

or, $AM \geq GM$ for $n = 2$.

Similarly considering only 2 observations x_3 and x_4, we have

$$\frac{x_3 + x_4}{2} \geq \sqrt{x_3 x_4}$$

If we consider two quantities $\frac{x_1 + x_2}{2}$ and $\frac{x_3 + x_4}{2}$

If we consider two quantities $\left(\frac{x_1+x_2}{2}\right)$ and $\left(\frac{x_3+x_4}{2}\right)$

then $$\frac{\frac{x_1+x_2}{2}+\frac{x_3+x_4}{2}}{2} \geq \sqrt{\left(\frac{x_1+x_2}{2}\right)\left(\frac{x_3+x_4}{2}\right)}$$

or, $$\frac{x_1+x_2+x_3+x_4}{4} \geq \sqrt{\sqrt{x_1 x_2}\sqrt{x_3 x_4}}$$

or, $$\frac{x_1+x_2+x_3+x_4}{4} \geq (x_1 x_2 x_3 x_4)^{1/4}$$

$\therefore$ AM $\geq$ GM for $n = 4$ observations.

Proceeding this way, it can be shown that AM $\geq$ GM whenever $n = 2$ or 4 or 8 or $\cdots 2^m$

ie $n = 2^m$, where m is a positive integer.

Let us suppose that the given value of n lies between two such values $2^{m-1} < n < 2^m$

Let $2^m = N$ (say) consisting of 'n' observations $x_1, x_2, \ldots, x_n$ and $(N-n)$ observations equal to A where $A = \frac{1}{n}\sum_{i=1}^{n} x_i$

$$\left.\boxed{\underbrace{x_1, x_2, \ldots, x_n}_{n \text{ terms}}, \underbrace{A, A, A, \ldots A}_{(N-n) \text{ terms}}}\right\} N \text{ terms.}$$

AM of these N observations is

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$$\frac{(x_1+x_2+\cdots+x_n)+A+A+\cdots+A}{N} = \frac{nA+(N-n)A}{N}$$

$$= \frac{\cancel{nA}+NA-\cancel{nA}}{N}$$

$$= A$$

GM of N observation is $\left(\underbrace{x_1 x_2 \cdots x_n}\underbrace{A A \cdots A}\right)^{1/N}$

$$= \left(G^n A^{N-n}\right)^{1/N}$$

For $N = 2^m$ observations $AM \geq GM$

or, $A \geq \left(G^n A^{N-n}\right)^{1/N}$

or, $A^N \geq G^n A^{N-n}$

or $A^{N-N+n} \geq G^n$

or $A^n \geq G^n$

or $A \geq G$ for $N = 2^m$ obs.
(any no. of obs)

④

Let us take the reciprocal of all 'n' observations

ie $\frac{1}{}$ $\frac{1}{}$ $\frac{1}{}$ $\cdots$ $\frac{1}{}$

ie $\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3} \dots \frac{1}{x_n}$

$$AM = \frac{\frac{1}{x_1}+\frac{1}{x_2}+\dots+\frac{1}{x_n}}{n} = \frac{1}{H} \quad \text{(from 3)} \quad (5)$$

And $GM = \left(\frac{1}{x_1} \frac{1}{x_2} \dots \frac{1}{x_n}\right)^{1/n}$

$$= \left(\frac{1}{G^n}\right)^{n}$$

$$GM = \frac{1}{G} \quad (6)$$

for any n obs $AM \geq GM$

$\frac{1}{H} \geq \frac{1}{G} \Rightarrow G \geq H \quad (7)$

$\therefore$ comparing (4) and (7)

$$\boxed{AM \geq GM \geq HM}$$ Proved

$AM \times HM = GM^2$

Let A, H and G be the respective AM, HM and GM of 2 observations, then,
(x_1 and x_2)

$$A = \frac{x_1 + x_2}{2}$$

$$G = (x_1 . x_2)^{1/2}$$

$$H = \frac{2}{\frac{1}{x_1} + \frac{1}{x_2}}$$

Now $A \times H = \left(\frac{x_1 + x_2}{\cancel{2}}\right) \times \left(\frac{\cancel{2}}{\frac{1}{x_1} + \frac{1}{x_2}}\right)$

$$A \times H = \frac{x_1 + x_2}{\frac{x_1 + x_2}{x_1 x_2}}$$

$$= \frac{\cancel{(x_1 + x_2)}\ x_1 . x_2}{\cancel{(x_1 + x_2)}}$$

$$A \times H = x_1 . x_2$$

$$A \times H = G^2$$

$\therefore$ for 2 obs $AM \times HM = GM^2$

$$\boxed{\sqrt{AM \times HM} = GM}$$

Combined mean or Pooled mean

Suppose n_1 observation for set 1 with mean $\bar{x}_1$

n_2 observation for set 2 with mean $\bar{x}_2$.

Then combined mean $\bar{X} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$

then combined mean $X = \frac{n_1 x_1 + n_2 x_2}{n_1 + n_2}$

or, $\bar{X} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{N}$

where $N = n_1 + n_2$.

Ex: The mean monthly salary paid to all employees in a certain company was Rs 500. The mean monthly salaries paid to male and female employees were 520 and 420 rupees respectively. Obtain the percentage of male to female employees in the company.

Soln: $\bar{X} = 500$ $\bar{x}_1 = 520$ $\bar{x}_2 = 420$

$n_1 = ?$ $n_2 = ?$

Using the formula $\bar{X} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$

or $500 = \frac{(n_1 \times 520) + (n_2 \times 420)}{n_1 + n_2}$

or, $500 n_1 + 500 n_2 = 520 n_1 + 420 n_2$

or, $20 n_1 = 80 n_2$

or, $\frac{n_1}{n_2} = \frac{80}{20} = \frac{4}{1}$

ie $n_1 : n_2 = 4 : 1$

$\therefore$ % of male employees $= \frac{4}{5} \times 100 = 80\%$

$\therefore$ % of female employees $= 20\%$

(ans)

Median $= x_l + \frac{N/2 - cf}{f_m} \times c$

x_l: lower boundary of median class

N: Total freq

cf: cumm freq preceding class

f_m: frequency of median class

c: width or class size of median class.

Mode $= x_l + \underbrace{\left(\frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1}\right)}_{} \times c$

x_l: lower limit of modal class

c: class size.

f_0 = frequency of modal class

f_{-1} = " preceding modal class

f_1 = " succeeding modal class

Calculate the median and mode from the following data

weekly	0-20	20-40	40-60	60-80	80-100

Weekly wages	0-20	20-40	40-60	60-80	80-100
No. of workers	40	51	64	38	7

Class boundary	Frequency	Cummulative Frequency
0-20	40	40
20-40	f_{-1} 51	91 Cf
40-60	f_0 64 $= f_m$	155
60-80	f_1 38	193
80-100	7	200 = N
	$N = \Sigma f = 200$	

median = 42.81 (ans).

Mode = 46.6 (ans)

Empirical formula: $\boxed{\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{Median})}$