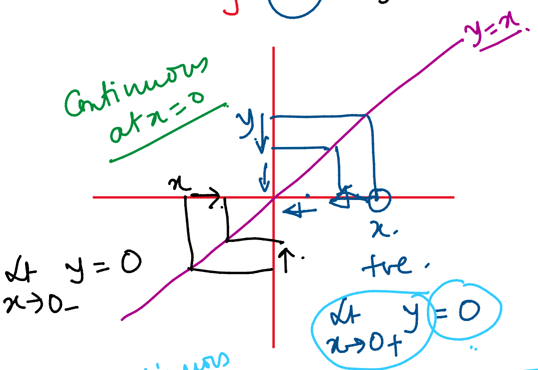
05 December 2023 20:00

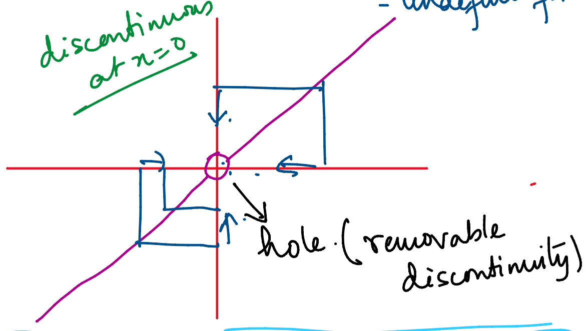
Limits

$y = x$ $\quad y(x=0) = 0$

$y = \frac{x^2}{x} \Rightarrow y = x, \; x \neq 0$

$= \text{undefined for } x = 0$





y is continuous at x=0

$\boxed{\lim_{x\to 0+} y = \lim_{x \to 0-} y = y(x=0)}$

RHL. LHL. functional value at x=0

$\lim_{x\to 0+} y = \lim_{x\to 0-} y = 0$

$y(x=0) \Rightarrow \text{undefined}$

y is discontinuous at x=0 ↓ (hole)

$y = \frac{x^2}{x}, \; x \neq 0$

$= 1, \; x = 0.$

$y = \frac{x^2 - 5x + 6}{x^2 - 4}$ → points of discontinuity.

$y = \frac{(x-2)(x-3)}{(x-2)(x+2)}$ $\qquad y = \frac{x-3}{x+2}, \; x \neq 2.$

$\boxed{= \text{undefined } \; x = 2.}$

hole.

→ +∞.

−2.00001

−2 −1.9999,

−∞.

$x = -2.00001$

$y = \frac{-5.00001}{-0.00001}$

$= 500001$

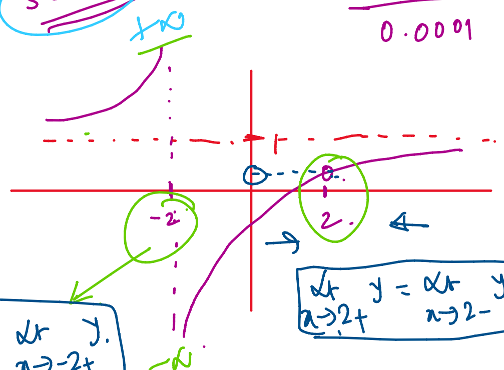
$y = \frac{x-3}{x+2}, \; x \neq 2.$

→ $x = -2$ $\quad y \Rightarrow \text{undefined}.$

$x = -1.9999.$

$y = \frac{-4.9999}{0.0001} = -49999$

−∞.



$\boxed{\lim_{x\to -2-} y \neq \lim_{x\to -2+} y.}$

⇓

limit doesnot exist.

$\boxed{\lim_{x\to 2+} y = \lim_{x \to 2-} y}$

$y = \frac{x-3}{x+2} = \frac{\frac{x-3}{x}}{\frac{x+2}{x}}$

$= \frac{1 - 3/x}{1 + 2/x}$

$\boxed{x \to -\infty}$ $\quad \frac{1}{x} \to 0,$ $\quad \boxed{x \to +\infty}$ $\quad \frac{1}{x} \to 0.$

$(y \to 1)$ $\qquad \boxed{y \to 1}$

$x \to -2^-$
$\Downarrow$
limit doesnot exist.

$\boxed{x \to -\infty}$ $\frac{1}{x} \to 0$, $\boxed{x \to +\infty}$ $\frac{1}{x} \to 0$.
$\boxed{y \to 1}$ $\boxed{y \to 1}$

$y = \frac{x-3}{x+2}$, $x \neq 2$. $\lim_{x \to 2} \frac{x-3}{x+2} = \frac{2-3}{2+2} = -\frac{1}{4}$

$y = \frac{x^2+x+1}{x^2+x+3}$ → Discriminant $= b^2-4ac$

ax^2+bx+c.

$D>0 \to$ 2 factors
$D=0 \to$ perfect square.
$D<0 \to$ no factors

$\lim_{x \to -1} y = \frac{(-1)^2+(-1)+1}{(-1)^2+(-1)+3}$

$= \frac{1}{3}$

$y = \frac{x^2+x+1}{x^2-1}$ $\lim_{x \to 1} y = \frac{3}{0} \Rightarrow \infty$.

$y = \frac{x^2-1}{x^2+x+1}$ $\lim_{x \to 1} y = 0$

$y = \frac{f(x)}{g(x)}$ $x \to a$, $f(a) = 0$, $g(a) = 0$.

$y = \frac{\sin x}{x}$. $x \to 0$, $y = \frac{0}{0}$.

① L Hospital's Rule..
② Taylor Series.

L Hospital's Rule

$\lim_{x \to a} \frac{f(x)}{g(x)}$ where $f(a) = 0$ & $g(a) = 0$
OR $f(a) = \pm\infty$ & $g(a) = \pm\infty$.

$\boxed{\frac{0}{0} \text{ OR } \frac{\infty}{\infty}}$

$f'(x) = \frac{d}{dx} f(x)$

$y = \frac{\sin x}{x}$ $f(x)$, $g(x)$

$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$ $g'(x) = \frac{d}{dx} g(x)$

$\lim_{x \to 0} y = \frac{0}{0}$ form

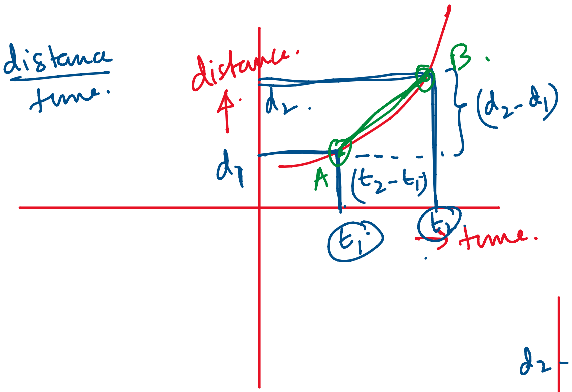
$\frac{d}{dx}(\sin x) = \cos x$.

$\frac{d}{dx}(x) = 1$

$= \lim_{x \to 0} \frac{\cos x}{1} = \cos 0 = 1$

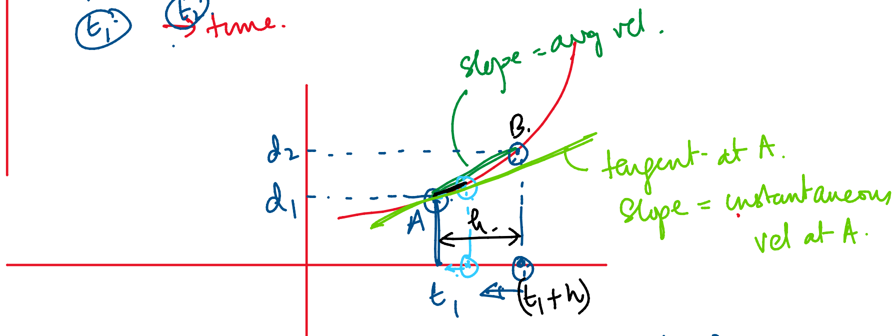
$\frac{d}{dx}(x) = 1$

$\text{Speed} = \frac{\text{distance}}{\text{time}}$



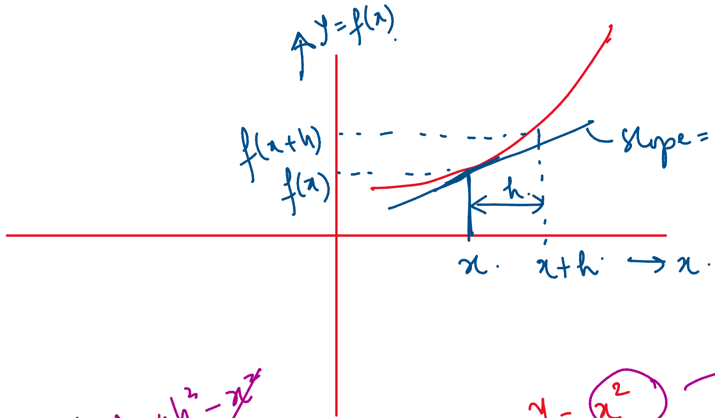
average speed between t_1 & t_2

$= \frac{d_2 - d_1}{t_2 - t_1} = \text{slope of } AB$



derivative of d w.r.t t at A.

$$\frac{d}{dt}(d) = \lim_{h\to 0} \frac{d(t_1+h) - d(t_1)}{h} = \text{slope of the tangent.}$$



$$\text{slope} = \frac{d}{dx} f(x) = \frac{d}{dx} y = f'(x) = y'$$

$$\boxed{f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}}$$

$\cancel{x^2} + 2hx + h^2 - \cancel{x^2}$

$y = x^2$

$$\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{2hx + h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= \lim_{h\to 0} (2x+h) = 2x$$

$y = x^n$

$$\lim_{h\to 0} \frac{(x+h)^n - x^n}{h}$$

$$= nx^{n-1}$$

$$\boxed{\frac{d}{dx} x^n = n x^{n-1}}$$

$$(a+b)^n = a^n + n a^{n-1} b + \frac{n(n-1)}{2} a^{n-2} b^2 + \dots + b^n$$

$$(x+h)^n = x^n + n x^{n-1} h + \frac{n(n-1)}{2} x^{n-2} h^2 + \dots + h^n$$

$$(x+h)^n - x^n = n x^{n-1} h + \frac{n(n-1)}{2} x^{n-2} h^2 + \dots + h^n$$

$$\frac{(x+h)^n - x^n}{h} = n x^{n-1} + \frac{n(n-1)}{2} x^{n-2} \cancel{h}^{\to 0} + \dots + \cancel{h^{n-1}}^{\to 0}$$