INEQUALITY 21.3.2024

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AM ≥ GM $\quad \frac{a+b+c}{3} \geq (abc)^{\frac{1}{3}}$

Sunday

$\frac{2557}{6}$ R → 1

$\frac{17}{6}$ → R → 5

$\frac{\alpha_1}{\alpha_2}$ → $\frac{\xi}{\psi}$ when $\xi < \psi$

Remainder → ξ

$a^2 + 2ab + b^2$

$\psi^2 + 2\xi\psi + \xi^2$

$\frac{5}{7}$ R → 5

$\frac{8}{7}$ 1

Numerator < Denominator

1

max value ~~>~~ ≤

union

$x \leq 3$ → max value 3

$x \geq 3$

3, 4, 5

#

$\frac{(1+x+x^2)(1+y+y^2)}{xy} \geq 9$

AM ≥ GM

≤ 9 < 9 > 9

$\frac{1+x+x^2}{x}$ a

$\frac{1+y+y^2}{y}$ b

AM ≥ GM

$\frac{a+b}{2} \quad (ab)^{\frac{1}{2}}$

$\frac{a+b+c+d}{4} \quad (abcd)^{\frac{1}{4}}$

a

$a_1 a_2 a_3$

$\frac{1}{x}, 1, x$ $\quad$ $1, y, \frac{1}{y}$

$\frac{\frac{1}{x}+1+x}{3} \geq \left(\frac{1}{x}\cdot 1 \cdot x\right)^{\frac{1}{3}}$ $\quad$ $\frac{1+y+\frac{1}{y}}{3} \geq \left(1\cdot y\cdot\frac{1}{y}\right)^{\frac{1}{3}}$

$\frac{1+x+x^2}{x} \geq 3$ $\quad$ $\frac{1+y+y^2}{y} \geq 3$

$\frac{1+x+x^2}{x}\cdot\frac{1+y+y^2}{y} \geq 9$

151 $a, b > 0$, $\boxed{a+b=1}$
then the least value of $\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)$

$\boxed{3/6/9/12}$

Ans: $\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right) = \frac{1+(a+b)+ab}{ab} = \frac{2+ab}{ab}$

$= 1+\frac{2}{ab}$

$\boxed{AM \geq GM}$ $\quad$ $\frac{a+b}{2} \geq \sqrt{ab}$

$\frac{1}{2} \geq \sqrt{ab}$

$\frac{1}{\sqrt{ab}} \geq 2$ $\quad$ $\boxed{\frac{1}{ab} \geq 4}$

$\boxed{1+\frac{2}{ab} \geq 9}$

$\boxed{\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right) \geq 9}$

$(x^2)^3 \to a^6 \to$

$(3+1) \to 4$

choosing of the number

$\boxed{a, b > 0}$

max value of $\frac{a^3 b^1}{(a+b)^4}$

4 value

max (C)

max value of $\dfrac{a^3b}{(a+b)^4}$

$\dfrac{a^3b^3}{}$

$\boxed{AM \geq GM}$

$3a, a, a, 3b$

a, a, a, a, b, b

Trick for AM GM question always take values in a way that power & coefficient both matches...

$2b \quad 2b$

$2, 4$

Hence we can

$\boxed{a, a, a, 3b}$

$$\frac{a+a+a+3b}{4} \geq \left(a^3 \cdot 3b\right)^{\frac{1}{4}}$$

$$\left[\frac{3(a+b)}{4}\right]^4 \geq \left[(3a^3b)^{\frac{1}{4}}\right]^4$$

$$\frac{27\,\cancel{81}\,(a+b)^4}{256} \geq \cancel{3}\,(a^3b)$$

$$\frac{27}{256} \geq \frac{a^3b}{(a+b)^4}$$

a^3b

$$\boxed{\frac{a^3b}{(a+b)^4} \leq \frac{27}{256}}$$

$a_n = \dfrac{10^{n+1}+1}{10^n+1} \qquad n \in N$

101

$10^n + 1$

$a_1 = \frac{10^2+1}{10^1+1} = \frac{101}{11}$

$a_2 = \frac{10^3+1}{10^2+1} = \frac{1001}{101}$

$\boxed{9062395123}$

Relation $a_n \to a_{n+1}$

$k^2 \times$

$10^n \cdot 10^2 = 10^2 \cdot k = 100k$

Let, $10^n = k$

$a_{n+1} - a_n = \frac{10^{n+2}+1}{10^{n+1}+1} - \frac{10^{n+1}+1}{10^n+1}$

$10^n = k > 0$

$= \frac{100k+1}{10k+1} - \frac{10k+1}{k+1}$

$= \frac{100k^2 + 101k + 1 - 100k^2 - 20k - 1}{(10k+1)(k+1)}$

$= \frac{81k}{(10k+1)(k+1)} > 0$

$81k > 0$, $(10k+1) > 0$, $(k+1) > 0$

$a_{n+1} - a_n > 0$

$a_{n+1} > a_n$

$a_{n+1} > a_n$

Increasing Sequence

$\boxed{a_4 > a_3 > a_2 > a_1}$

$n^2 \quad 1^2 \quad 2^2 \quad 3^2 \qquad \frac{1}{n}$

$U_n = \frac{4+n}{n} = \left(\frac{4}{n} + 1\right)$

$U_n = \frac{4+n}{n}$

$U_1 = \frac{4}{1}+1 = 5 \rangle$ 2

$U_2 = \frac{4}{2}+1 = 3 \rangle$ 0.67

$U_3 = 4/3 + 1 = 2.33 \rangle$ 0.33

$U_4 = 2 \rangle$ 0.22

$U_5 = 1.8 \rangle$ 0.13

$U_6 = 1.\cancel{8}67$ 0.10

Amount of fall is also falling.

$U_7 = \frac{4}{7}+1 = 1.57 \rangle$ 0.07

$\frac{4}{8}+1 = 1.50$ $\boxed{0.007}$

$U_n = \frac{4}{n}+1$

$U_\infty = \frac{4}{\infty}+1 = 1$

CMI

HW

$a, b > 0 \quad n \in N$

$M_n = \frac{a^n + b^n}{2} \qquad M_1 = \frac{a+b}{2}$

$\boxed{M_n^1 \le M_1^n \;/\; M_n^1 = M_1^n \;/\; M_n^1 \geq M_1^n \;/\; \text{None}}$

$a_n = (1^\alpha + 2^\alpha + \dots + n^\alpha)^n$

$b_n = n^n (n!)^\alpha$

$\boxed{1.2.3 \dots n = n!}$

$AM \geq GM$

$1^\alpha + 2^\alpha + \dots + n^\alpha \geq (1^\alpha . 2^\alpha . 3^\alpha \dots n^\alpha)^{\frac{1}{n}}$

$$\frac{1^{\alpha}+2^{\alpha}+3^{\alpha}+\dots+n^{\alpha}}{n} \geq \left(1^{\alpha}\cdot 2^{\alpha}\cdots n^{\alpha}\right)^{\frac{1}{n}}$$

$$\geq \left(n!\right)^{\frac{\alpha}{n}}$$

$$\boxed{a_n \geq b_n}$$

$$a_n > b_n \Rightarrow a_n \geq b_n$$

$4 \quad \boxed{6 \quad 7 \quad 9} \quad 10$

$$\rightarrow 4 < x < 10$$

$$\boxed{5 < x < 7}$$

In case both & both answers are same then the smaller interval with precise values will be accepted.

$\boxed{AM \geq GM \geq HM}$

Arith Geometric Harmonic mean

Theory

AM $\frac{a+b+c}{3}$

GM $(abc)^{\frac{1}{3}}$

HM $\frac{3}{\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)}$

HM $\frac{2ab}{a+b}$

$$\frac{2}{\frac{1}{a}+\frac{1}{b}}$$

$$\boxed{a,b,c>0}$$

$$A = \frac{b^2+c^2}{b+c} + \frac{c^2+a^2}{c+a} + \frac{a^2+b^2}{a+b}$$

then, $0 \leq A < a$

$a+b \leq A < a+b+c$

$a \leq A < a+b$

$a+b+c \leq A < 2(a+b+c)$

$\boxed{9062395123}$

Using HM - AM Inequality

$\boxed{\text{Hint}}$ $\frac{a+b}{2} \geq \frac{1}{\frac{\frac{1}{a}+\frac{1}{b}}{2}}$

$b>0,\ d>0$ $\boxed{\frac{a}{b} < \frac{c}{d}}$

HW

then

a) $\frac{a}{b} < \frac{a-c}{b-d} < \frac{c}{d}$

b) $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$

c) $\frac{a}{b} < \frac{a-c}{b+d} < \frac{c}{d}$

d) $\frac{a}{b} < \frac{a+c}{b-d} < \frac{c}{d}$