

AM ≥ GM

$$\frac{a+b+c}{3} \geq (abc)^{\frac{1}{3}}$$

Sunday

$$\frac{2557}{6}$$

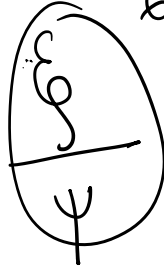
$$R \rightarrow 1$$

$$\frac{17}{6} \rightarrow R \rightarrow 5$$

$$\frac{x_1}{x_2}$$

$$a^2 + 2ab + b^2$$

$$\psi^2 + 2\psi\phi + \phi^2$$



when $\phi < \psi$

Remainder $\rightarrow \phi$

$$\frac{5}{7} \div \frac{5}{5} \rightarrow R \rightarrow 5$$

$$\frac{8}{7}$$

①

numerator < denominator

Max Value $\rightarrow \leq$
Min $\rightarrow \geq$
 $x \leq 3 \rightarrow$ max value ③
 $x \geq 3$
③, 4, 5
AM ≥ GM

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$$\frac{(1+x+x^2)(1+y+y^2)}{xy}$$

$$\leq 9$$

$$< 9$$

$$\geq 9 > 9$$

$$\frac{1+x+x^2}{x} \quad a$$

$$\frac{1+y+y^2}{y} \quad b$$

$$\frac{a+b}{2} \geq \sqrt{ab} \quad (ab)^{\frac{1}{2}}$$

$$\frac{a+b+c+d}{4} \geq \sqrt[4]{abcd} \quad (abcd)^{\frac{1}{4}}$$

$$1, x, \frac{1}{x}$$

$$1, n, \frac{1}{n}$$

a
 a_1, a_2, a_3
 $x, 1, x$
 $\frac{1}{x} + 1 + x \geq \left(\frac{1 \cdot 1 \cdot x}{x \cdot 1 \cdot x}\right)^{\frac{1}{3}}$
 $\frac{1+y+y}{3} \geq \left(\frac{1 \cdot y \cdot \frac{1}{y}}{3}\right)^{\frac{1}{3}}$
 $\frac{1+x+x}{3} \geq 3$
 $\frac{1+y+y}{3} \geq 3$
 $\frac{1+x+x^2}{3} \geq 3$
 $\frac{1+y+y^2}{3} \geq 9$

ISI
 $a, b > 0$
 then the least value of $(1 + \frac{1}{a})(1 + \frac{1}{b})$
 3/6/9/12

Ans: $(1 + \frac{1}{a})(1 + \frac{1}{b})$
 $= \frac{1 + (a+b) + ab}{ab} = \frac{2+ab}{ab}$
 $= 1 + \frac{2}{ab}$

AM \geq GM
 $\frac{a+b}{2} \geq \sqrt{ab}$
 $\frac{1}{2} \geq \sqrt{\frac{1}{ab}}$
 $\frac{1}{\sqrt{ab}} \geq 2$
 $\frac{1}{ab} \geq 4$

$1 + \frac{2}{ab} \geq 9$

$(1 + \frac{1}{a})(1 + \frac{1}{b}) \geq 9$

choosing of the number

$a, b > 0$
 $a=3, b=1$
 $(a+b) \geq 4$
 $(x^2)^3 \rightarrow a^6$
 $3+1 \rightarrow 4$
 4 value

max \leq

map value of $\frac{a^3}{(a+b)^4}$

AM ≥ GM

There for AM GM function always take values in a way that power & coefficient both matches...
 $3a, a, a, 3b$
 a, a, a, a, b, b
 $(2b)^{2b}$
 $(2)^n (4)$

Hence we can

$$\frac{a+a+a+3b}{4} \geq \sqrt[4]{\frac{a^3 \cdot 3b}{4}}$$

$$\left(\frac{3(a+b)}{4}\right)^4 \geq (3a^3b)^4$$

$$\frac{27 \cdot 81}{256} (a+b)^4 \geq 3 (a^3b)^4$$

$$\frac{27}{256} \geq \frac{a^3b}{(a+b)^4}$$

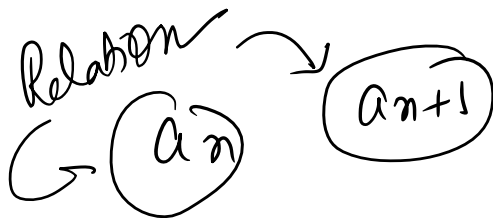
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$$a_n = \frac{10^{n+1} + 1}{10^n + 1} \quad n \in \mathbb{N}$$

→ 101

$$a_1 = \frac{10^1 + 1}{10^1 + 1} = \frac{101}{11}$$

$$a_2 = \frac{10^3 + 1}{10^2 + 1} = \frac{1001}{101}$$



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$$k^2 \times 10^n \cdot 10^2 = 10^{n+2} \cdot k^2$$

Let, $10^n = k$

$$a_{n+1} - a_n = \frac{10^{n+2} + 1}{10^{n+1} + 1} - \frac{10^{n+1} + 1}{10^n + 1}$$

$10^n = k > 0$

$$= \frac{100k + 1}{10k + 1} - \frac{10k + 1}{k + 1}$$

$$= \frac{100k^2 + 10k + 1 - 10k^2 - 20k - k}{(10k + 1)(k + 1)}$$

$$= \frac{81k^2 - 20k - k}{(10k + 1)(k + 1)} > 0$$

$a_{n+1} - a_n > 0$
 $a_{n+1} > a_n$

Increasing Sequence

$a_4 > a_3 > a_2 > a_1$

$k^2 \quad 1^2 \quad 2^2 \quad 3^2 \quad \frac{1}{n}$

$U_n = \frac{4+n}{n} = \left(\frac{4}{n} + 1\right)$

$U_n = \frac{4+n}{n} = 1 + \frac{1}{n}$

Amount of fuel is also falling.

$U_1 = \frac{4}{1} + 1 = 5 \rightarrow 2$

$U_2 = \frac{4}{2} + 1 = 3 \rightarrow 0.67$

$U_3 = \frac{4}{3} + 1 = 2.33 \rightarrow 0.33$

$U_4 = 2 \rightarrow 0.22$

$U_5 = 1.8 \rightarrow 0.13$

$U_6 = 1.67 \rightarrow 0.10$

$U_7 = \frac{4}{7} + 1 = 1.57 \rightarrow 0.07$

$U_8 = \frac{4}{8} + 1 = 1.50 \rightarrow 0.007$

$U_n = \frac{4}{n} + 1$

$U_\infty = \frac{4}{\infty} + 1 = 1$

AME
HW

$a, b > 0$

$n \in \mathbb{N}$

$M_n = \frac{a^n + b^n}{2}$

$M_1 = \frac{a+b}{2}$

$M_n^1 \leq M_n^2$ / $M_n^1 = M_n^2$ / $M_n > M_1^2$ / None



$a^n = (1^\alpha + 2^\alpha + \dots + n^\alpha)^n$

$b^n = n^n (n!)^\alpha$

$1 \cdot 2 \cdot 3 \dots n = n!$

AM $>$ HM

$\alpha \dots \alpha \dots + n^\alpha \rightarrow (1^\alpha \cdot 2^\alpha \cdot 3^\alpha \dots n^\alpha)^\alpha$

$$\frac{1^\alpha + 2^\alpha + 3^\alpha + \dots + n^\alpha}{n}$$

$$\approx (1.2)^n$$

$$\approx (n!)^\alpha \cdot n^n$$

$$a_n > b_n$$

$$\Rightarrow a_n > b_n$$

$$\rightarrow a_n > b_n$$



In case both are same then the smaller interval with freer values will be accepted.

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$$AM \geq GM \geq HM$$

Arithmetic mean Geometric mean Harmonic mean

Theory

AM $\frac{a+b+c}{3}$

GM $(abc)^{\frac{1}{3}}$

HM $\frac{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}{3}$

AM $\frac{\frac{2ab}{a+b}}{2}$

$$a, b, c > 0$$

$$A = \frac{b^2 + c^2}{b+c} + \frac{c^2 + a^2}{c+a} + \frac{a^2 + b^2}{a+b}$$

then,

$$0 \leq A \leq ca$$

$$a+b \leq A \leq ca+bc$$

$$a \leq A \leq a+b$$

$$a+b+c \leq A \leq 2(a+b+c)$$

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0 Sims HM-AM inequality
 $\frac{a+b}{2} \geq \frac{1}{\frac{1}{a} + \frac{1}{b}}$

Hint

$b > 0, d > 0$

$$\frac{a}{b} < \frac{c}{d}$$

HW

then

- a) $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$
- b) $\frac{a}{b} < \frac{a-c}{b-d} < \frac{c}{d}$
- c) $\frac{a}{b} < \frac{a-e}{b-d} < \frac{c}{d}$
- d) $\frac{a}{b} < \frac{a+c}{b-d} < \frac{c}{d}$