08 January 2024 10:47 PM

Q The equilibrium condition for three related market is given by

$$11P_1 - P_2 - P_3 = 31$$

$$-P_1 + 6P_2 - 2P_3 = 26$$

$$-P_1 - 2P_2 + 7P_3 = 24$$

Using the inverse matrix method find P_1, P_2 and P_3.

Let us write the system of equation in matrix form.

$$\begin{bmatrix} 11 & -1 & -1 \\ -1 & 6 & -2 \\ -1 & -2 & 7 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 31 \\ 26 \\ 24 \end{bmatrix}$$

$(-1\times 7) - (-1\times -2)$

$-7 - 2$

$= -9$

$$|A| = \begin{vmatrix} 11 & -1 & -1 \\ -1 & 6 & -2 \\ -1 & -2 & 7 \end{vmatrix} = 11\left(\boxed{42-4}\right) + 1\left(\boxed{-7-2}\right) - 1(2+6)$$

$$= 11(38) - 9 - 8$$

$$= 418 - 17$$

$$= 401$$

Cofactors are

$$C = \begin{bmatrix} a_{11} = 38 & a_{12} = +9 & a_{13} = 8 \end{bmatrix}$$

$$\begin{bmatrix} & & \\ a_{21} = +9 & a_{22} = 76 & a_{23} = +23 \\ & & \\ a_{31} = 8 & a_{32} = +23 & a_{33} = 65 \end{bmatrix}$$

$$\text{adj}(A) = C' = \begin{bmatrix} 38 & 9 & 8 \\ 9 & 76 & 23 \\ 8 & 23 & 65 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{401}\begin{bmatrix} 38 & 9 & 8 \\ 9 & 76 & 23 \\ 8 & 23 & 65 \end{bmatrix}$$

Now $AX = B$

or $X = A^{-1}B$

or $$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} \frac{28}{401} & \frac{9}{401} & \frac{8}{401} \\ \frac{9}{401} & \frac{76}{401} & \frac{23}{401} \\ \frac{8}{401} & \frac{23}{401} & \frac{65}{401} \end{bmatrix} \begin{bmatrix} 31 \\ 26 \\ 24 \end{bmatrix}$$

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 3.98 \\ 6.98 \\ 5.99. \end{bmatrix}$$

① Law of matrix (Associative Distributive) etc

② Cofactors and principal minors

A^{-1} ... us equation A^{-1}

$A = [1\ 2]$

② Cofactor ...

③ A^{-1}

④ solve: Simultaneous equations using A^{-1}

⑤ Cramer's Rule.

$A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 3 & 2 \\ 6 & 5 \end{bmatrix}$

AB

$\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 6 & 5 \end{bmatrix}$

$[1\times3+2\times6$