STATISTICS Session...

Law of Large numbers + Inequalities + CGF

CF.

Random variable

$$p(x) = \frac{1}{2^x}$$

MGF

$$M_X(t) = E(e^{tX}) = \sum e^{tx} p(x)$$

$$\Rightarrow \sum \left(\frac{e^{tx}}{2^x}\right) = \sum \left(\frac{e^t}{2}\right)^x = \frac{e^t}{2} + \left(\frac{e^t}{2}\right)^2 + \left(\frac{e^t}{2}\right)^3 + \dots$$

$$\Rightarrow \frac{e^t/2}{1 - e^t/2} = \frac{e^t}{2 - e^t} \Rightarrow \text{MGF}$$

$$\text{Mean} = \mu_1' = \left.\frac{\partial M(t)}{\partial t}\right|_{t=0}$$

$$\mu_2' = \left.\frac{\partial^2 M(t)}{\partial t^2}\right|_{t=0}$$

$$\mu_2 = \mu_2' - \mu_1'^2 =$$

Please note fractional division...

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$$f(x) = \begin{cases} x & 0 \le x < 1 \\ 2-x & 1 \le x < 2 \\ 0 & \text{o.w.} \end{cases}$$

$$\begin{cases} 2-x & 1 \le x < 2 \\ 0 & \text{o.w.} \end{cases}$$

$\mu_1', \mu_2, \mu_3, \mu_4$

$$M_X(t) = E(e^{tX}) = \int_0^1 x\, dx + \int_1^2 (2-x)\, dx$$

$$\Rightarrow \left(\frac{e^t - 1}{t}\right)^2$$

Cumulant Val

$$K_X(t) = \ln M_X(t) = 2\log\left[\frac{\left(1+t+\frac{t^2}{2!}+\frac{t^3}{3!}+\dots\right)-1}{t}\right]$$

$$= 2\ln\left[1+\left(\frac{t}{2}+\frac{t^2}{3!}+\dots\right)\right]$$

$$= 2\left[\left(\frac{t}{2}+\frac{t^2}{3!}+\dots\right)-\frac{1}{2}\left(\frac{t}{2}+\frac{t^2}{3!}+\dots\right)^2 + \frac{1}{3}\left(\frac{t}{2}+\frac{t^2}{3!}+\dots\right)^3 - \frac{1}{4}\left(\frac{t}{2}+\frac{t^2}{3!}+\dots\right)^4 + \dots\right]$$

Thu 5:30 PM 28th

Mean $= K_1 =$ Coeff of t in $= 2 \cdot \frac{1}{2} = 1$

$$\mu_2 = \kappa_2 = \text{Coeff } \frac{t^2}{2!} = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

r^{th} cumulant of a distribution

$$f(x) = ce^{-cx} \quad 0 \le x < \infty$$

$$M_X(t) = E(e^{tX}) = \int_0^\infty e^{tx} \cdot e^{-cx}\, dx$$

$$M_X(t) = c\int_0^\infty e^{-(c-t)x}\, dx$$

$$\Rightarrow \left(1-\frac{t}{c}\right)^{-1}$$

$$\Rightarrow (1 - t/c)^{-1}$$

$\boxed{\text{CGF}}$

$$K_X(t) = \ln M_X(t) = -\ln(1 - t/c)$$

$$= t/c + \frac{(t/c)^2}{2} + \frac{(t/3)^3}{3} + \dots + \frac{(t/c)^r}{r} + \dots$$

Small $\boxed{\text{Kappa}}$

κ_r

ψ $\boxed{\text{PSI}}$

~~Trident~~ Ψ

$a^2 + 2ab + b^2$

$\psi^2 + 2\psi\omega + \omega^2$

Ξ ξ Ω ω Δ ∇ Nabla

ν

Ξ Θ theta θ

$0 \rightarrow$ Omicron $\infty \rightarrow$ Lemniscate

$$\kappa_r = r^{th} \text{ Cumt} = \text{Coef } \frac{t^r}{r!} \text{ in } K_X(t)$$

$$= \frac{1.2.3\dots(r-1)}{c^r} = \frac{(r-1)!}{c^r}$$

Chebychev's
Markov's Inequality

$$P(\mu - k\sigma \le X \le \mu + k\sigma) = P(|X - \mu| \le k\sigma) \ge 1 - \frac{1}{k^2}$$

k = Cnbre

Find the least value of K

μ, σ^2

Find the least value of (K) for which the probability that the R.V. X (μ, σ^2) taken a value b/w $\mu \pm k\sigma$ is ≥ 0.95 & ≥ 0.99

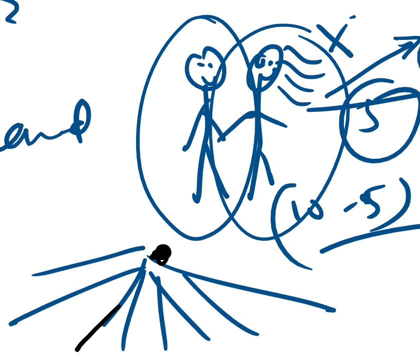
$1 - \frac{1}{k^2} = 0.95$

$\frac{1}{k^2} = 5/100$ $\quad k^2 = 20 \quad k = 5\sqrt{2}$ — $k > 0$

$1 - \frac{1}{k^2} = 0.99 \quad \frac{1}{k^2} = \frac{1}{100} \quad \boxed{k = 10}$ $\quad K > 0$

Durga Puja

Prefland



$5 - 11$

$0 < x < 5$

5

$(10 - 5)$

K

die → dice

X = Sum → 2, 12

Upper Bound

$P(X > 11)$

$P(|X - 7| > 4)$

$P(X > 4 + 7 \text{ or } X < 7 - 4)$

$P(X > 11 \text{ or } X < 3)$

$P(X = 12) + P(X = 2)$

$\frac{1}{\ } + \frac{1}{\ } = 2/\ldots = 1/18$

Actual Probability

the Probability

$P(X=12) + P(X=2)$
$= \frac{1}{36} + \frac{1}{36} = 2/36 = 1/18$

$= 0.055$

Chebyshev's Value

$$P\left[|X-\mu_X| > k\right] \le \frac{\operatorname{var}(X)}{k^2}$$

$E(X_1) = \frac{1}{6}(1+2+\dots 6) = 7/2 = 3.5$

$E(X_1^2) = \frac{1}{6}(1^2+2^2+\dots 6^2) = 91/6$

$V(X) = 91/6 - 49/4 = 35/12$

$X = X_1 + X_2$ $\quad \mu(X) = E(X_1 + X_2) = 7/2 \cdot 2 = 7$

$V(X) = V(X_1) + V(X_2) = 35/12 \cdot 2 = 35/6$

$$P\left[|X-7| \ge k\right] \le \frac{35}{6k^2}$$

$$P\left[|X-\mu_X| > k\right] \le \frac{35}{6 \cdot 16} = 35/96$$

$= 0.36$

As not always the values are known in the system

So, with limited analysis we sometimes can't calculate..

↓

Here, C.B. Inequality is useful

useful ..

(5) 2016

$f(x) = e^{-x} \quad x \geq 0$ diff from actual prob ??

$P(|X-1| > 2) < ?$

Imd vs ANS

Rajasthan

100

Indian

$100 > 10$

$\ln 100 > \ln 10$

$2 \ln 10 > \ln 10$

$2 > 1$

$$E(X^r) = \int_0^\infty x^r e^{-x}\,dx$$

$$= \Gamma(r+1) = r!$$

$\Gamma(3) = 2!$

Note Beta/Gamma function

$E(X) = 1 \quad E(X^2) = 2! = 2$

$V(X) = 1$

$$P\left[|X - E(X)| > K\right] \leq \frac{\operatorname{var} X}{K^2}$$

$$P\left[|X-1| > K\right] < \frac{1}{K^2}$$

Taking $K = 2$, $\quad P\left[|X-1| > 2\right] < \frac{1}{4}$

Actual

$$P\left[|X-1| > 2\right] = 1 - P\left[|X-1| \leq 2\right]$$

$$= 1 - P\left[1-2 \leq X \leq 1+2\right]$$

$$= 1 - P\left[-1 \leq X \leq 3\right]$$

$$= 1 - P(0 \leq X \leq 3) \quad \boxed{X > 0}$$

$$= 1 - \int_0^3 e^{-x}\,dx = 1 - \left|e^{-x}\right|_0^3$$

$$= 1 - \int_0 e^{-x}dx = 1 - |e^{-x}|_0$$

$$= 1-(1-e^{-3})$$

$$= \boxed{e^{-3}}$$

Sum

$E(x^2) < \infty$

then prove that $P\{|X| \geq a\} \leq \frac{1}{a^2}E(X^2)$

$\boxed{a > 0}$

Use CB inequality to show for $n > 36$ the prob that in 3 throws of a fair die, the number of sixes lies between $\frac{1}{6}n \pm \sqrt{n}$ is at least $31/36$

Ans: $X \sim B\left(n, p = 1/6\right)$

$E(X) = \mu_X = np = \frac{n}{6}$

$$\sigma_X^2 = V(X) = np(1-p) = n \cdot \frac{1}{6} \cdot \frac{5}{6} = \frac{5n}{36}$$

By CB inequality $P\left[|X - \mu_X| < K\right] \geq 1 - \frac{Var(X)}{K^2}$, $K > 0$

$$P\left[\left|X - \frac{n}{6}\right| < K\right] \geq 1 - \frac{5n}{36K^2}$$

$$P\left[\frac{n}{6} - K < X < \frac{n}{6} + K\right] \geq 1 - \frac{5n}{36K^2}$$

chang $K = \sqrt{n}$ $\longrightarrow$ $P\left[\frac{n}{6} - \sqrt{n} < X < \frac{n}{6} + \sqrt{n}\right] \geq 1 - 5/36$

$= 31/$

$\xrightarrow{}$ $P\left[\frac{n}{6} - \sqrt{n} < X < \frac{n}{6} + \sqrt{n}\right] \geq 1 - \frac{5}{36}$

$= \frac{31}{36}$

6's obtained in n tosses of ~~fair~~ dice cannot be -ve

We must have, $\frac{n}{6} - \sqrt{n} \geq 0$

$$\sqrt{n}\left(\sqrt{n} - 6\right) \geq 0$$

$$\sqrt{n} \geq 6$$

$$n \geq 36$$

$$P\left[\frac{n}{6} - \sqrt{n} < X < \frac{n}{6} + \sqrt{n}\right] \geq \frac{31}{36}$$