Special type of questions on No system & functions

P(x) is a polynomial of degree 98 such that $P(K) = \frac{1}{K}$ for K = 1, 2, 3, ..., 99. The value of P(100) is

(A) $\lfloor 100 + 1$ (B) $\frac{1}{100}$ (C) $\frac{1}{50}$ (D) $\frac{1}{\lfloor 100}$

Factor theorem.
If $f(x) = 0$ for $x = a$
then $(x-a)$ is a factor of $f(x)$

$P(x) = \frac{1}{x}$ for $\boxed{x = 1, 2, \ldots 99.}$

$x P(x) = 1$

$f(x) = \boxed{xP(x) - 1} = 0$ for $x = 1, 2, \cdots 99.$

$f(x) = 0$ for $x = 1, 2, \cdots 99.$

$\Rightarrow (x-1), (x-2), \ldots (x-99)$ are factors of $f(x)$

$f(x) = C(x-1)(x-2) \ldots (x-99)$

$xP(x) - 1 = C(x-1)(x-2) \ldots (x-99)$

put $x=0$.

$-1 = C(-1)(-2) \ldots (-99)$

$-1 = (-1)^{99} C \cdot 99!$

$C = \frac{1}{99!}$

$xP(x) - 1 = \frac{1}{99!}(x-1)(x-2) \ldots (x-99)$

$x = 100$.

$100P(100) - 1 = \frac{1}{99!}(99 \cdot 98 \ldots 1) = \frac{1}{\cancel{99!}} \cdot \cancel{99!} = 1$

$100P(100) = 2.$

$P(100) = \frac{2}{100} = \frac{1}{50}$

If x and y both are non-negative integral values for which $(xy - 7)^2 = x^2 + y^2$, then find the sum of all possible value of x

(A) 7 (B) 10 (C) 13 (D) 14

$x, y \in W$

Trick express as difference of 2 squares which equates to an integer

$2xy + x^2y^2 - 14xy + 49 = x^2 + y^2 + 2xy.$

$x^2y^2 - 12xy + 49 = (x+y)^2.$

$(x^2y^2 - 12xy + 36) + 13 = (x+y)^2$

$(xy-6)^2 + 13 = (x+y)^2$

$(x+y)^2 - (xy-6)^2 = 13.$ $\Rightarrow (x+y+xy-6)(x+y-xy+6) = 13$
1×13

$x+y+xy-6 = 13.$
$x+y-xy+6 = 1$

$2(x+y) = 14$

$\boxed{x+y = 7}$

x	y
0	7
1	6
2	5
3	4
4	3
5	2

$(xy-7)^2 = x^2 + y^2$

4 3
5 2
6 1
7 0

$(x+y)^2 - (xy-6)^2 = 13.$

$(xy-7)^2 = x^2+y^2$

1×13

Sum $= 0+3+4+7 = 14$

Let $x = \sqrt{3-\sqrt{5}}$ and $y = \sqrt{3+\sqrt{5}}$. If the value of the expression $x - y + 2x^2y + 2xy^2 - x^4y + xy^4$ can be expressed in the form $\sqrt{p} + \sqrt{q}$ where $p, q \in N$, then $(p + q)$ has the value equal to

(A) 410 (B) 610 (C) 510 (D) 540

$3+\sqrt{5} = \frac{1}{2}(6+2\sqrt{5}) = \frac{1}{2}(\sqrt{5}+1)^2$ $y = \sqrt{3+\sqrt{5}} = \frac{1}{\sqrt{2}}(\sqrt{5}+1) = \frac{\sqrt{2}(\sqrt{5}+1)}{2}$

$3-\sqrt{5} = \frac{1}{2}(\sqrt{5}-1)^2$ $x = \frac{\sqrt{2}(\sqrt{5}-1)}{2}.$

$(x-y) + 2xy(x+y) - xy(x^3-y^3) = (x-y) + 2xy(x+y) - xy[(x-y)^3 + 3xy(x-y)]$

$x+y = \frac{\sqrt{2}}{2} \cdot 2\sqrt{5} = \sqrt{10}$ $xy = \frac{2}{4}(5-1) = 2.$

$x-y = \frac{\sqrt{2}}{2}(-2) = -\sqrt{2}.$

$= -\sqrt{2} + 4\sqrt{10} - 2[-2\sqrt{2} + 6(-\sqrt{2})]$

$= -\sqrt{2} + 4\sqrt{10} + 4\sqrt{2} + 12\sqrt{2} = 15\sqrt{2} + 4\sqrt{10}$

$= \sqrt{450} + \sqrt{160} = \sqrt{p} + \sqrt{q}$

$p+q = 610$

Consider three natural numbers x, y and z such that when divided by 5, 7 and 11 respectively each leave remainder k. Then the expression $xyz - k(xy + yz + zx) + k^2(x + y + z) - k^3$ is necessarily divisible by

(A) 35 (B) 77 (C) 55 (D) 385

More than 1 answer is correct.

$x = 5a + k$ $y = 7b + k$ $z = 11c + k.$

$0 \le k \le 4$

$k = 0$

$x = 5a$ $y = 7b$ $z = 11c.$

$xyz = 5 \times 7 \times 11\, abc = 385\, abc.$

(C) 35 (D) 385

Let P(x) be a polynomial such that x. P (x − 1) = (x − 4) P (x) ∀x ∈ R. Then the degree of P(x) cannot be

(A) 2 ✓ (B) 3 ✓ (C) 4 (D) 5 ✓

More than one answer

$$xP(x-1) = (x-4)P(x)$$

factors → for what value of x $P(x) = 0$

$x=0$ $\quad 0 = -4P(0) \Rightarrow \boxed{P(0)=0} \Rightarrow x$ is a factor of $P(x)$

$x=1$ $\quad 1\cdot\underbrace{\boxed{P(0)}}_{=0} = -3P(1) \qquad 0 = -3P(1) \Rightarrow \boxed{P(1)=0} \Rightarrow (x-1)$ is a factor.

$x=2$ $\quad 2\underbrace{\boxed{P(1)}}_{=0} = -2P(2) \Rightarrow \boxed{P(2)=0} \Rightarrow (x-2)$ is a factor.

$x=3$ $\quad 3\cdot\underbrace{\boxed{P(2)}}_{=0} = -P(3) \Rightarrow P(3) = 0 \Rightarrow (x-3)$ is a factor.

$x=4$ $\quad 4\cdot P(3) = 0 \Rightarrow 0 = 0$

$$P(x) = Ax(x-1)(x-2)(x-3)$$

$\Downarrow$

degree $= 4$

Find the first digit from left of 3^{62} where log 3 = 0.4771

$\neq 3$

$\log(3^{62}) = 62 \log 3 = 62 \times 0.4771 = \underline{29.5802}$

$\log 2 = 0.3010.$

$\log 3 = 0.4771.$

$\log 7 = 0.8451$

$\log(3^4) = 4\log 3 = 4 \times 0.4771 = 1.9084$

$\log 4 = 0.602.$

$\log 5 = \log\left(\frac{10}{2}\right)$

$= 1 - \log 2$

$= 1 - 0.301 = 0.699$

$\log 3^2 = 0.$

No of digits of $3^4 = 1+1 = 2.$

$0.9084 < 0.9542 = \log 9.$

8

$0.5802 < 0.602 = \log 4.$

$\Downarrow$

3

More than 1 correct.

Let x, y, z be three positive real numbers such that

x+[y]+{z} = 13.2

[x]+{y}+z = 14.3

{x}+y+[z] = 15.1

where [a] denotes the greatest integer ≤ a and {b} denotes the fractional part of b, then

(A) xyz = 349.32 (B) x + y + z = 21.3

(C) x + y - z = 4.9 (D) x - y + z = 27.6

$\boxed{x = [x] + \{x\}}$

$x = 2.3975$

$[x] = 2 \quad \{x\} = 0.3975$

$x+y+z = 21.3$

$-\{[x]+\{y\}+z = 14.3\}$

$\{x\} + [y] = 7$

$\boxed{[y] = 7 \quad \{x\} = 0}$

$x + [y] + \{z\} = 13.2$

$[x] + \{y\} + z = 14.3$

$\{x\} + y + [z] = 15.1$

$2x + 2y + 2z = 42.6$

$x + y + z = 21.3$

$x+y+z = 21.3$

$-[x + [y] + \{z\} = 13.2]$

$\{y\} + [z] = 8.1$

fr. Int

$\boxed{[z] = 8 \quad \{y\} = 0.1}$

$\{x\} = 0$

$13.1 - 8.2 = 4.9$

$x = 6 \quad z = 8.2$

$y = 7.1$

$x + y + z = 21.3$

$\{z\} + y + [z] = 15.1$

$[x] + \{z\} = 6.2$

$xyz = 349.32$

$\boxed{[x] = 6 \quad \{z\} = 0.2}$