

Factor theorem
 If $f(x) = 0$ for $x = a$
 then $(x-a)$ is a factor
 of $f(x)$

$P(x) = \frac{1}{x}$ for $x = 1, 2, \dots, 99$.

$x P(x) = 1$

$f(x) = x P(x) - 1 = 0$ for $x = 1, 2, \dots, 99$.

$f(x) = 0$ for $x = 1, 2, \dots, 99$.

$\Rightarrow (x-1), (x-2), \dots, (x-99)$ are factors of $f(x)$

$f(x) = C(x-1)(x-2) \dots (x-99)$

$x P(x) - 1 = C(x-1)(x-2) \dots (x-99)$

put $x=0$.

$-1 = C(-1)(-2) \dots (-99)$

$-1 = (-1)^{99} C \cdot 99!$

$C = \frac{1}{99!}$

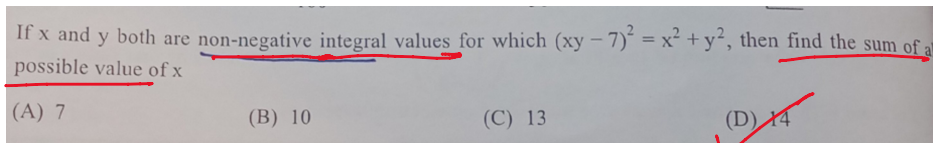
$x P(x) - 1 = \frac{1}{99!} (x-1)(x-2) \dots (x-99)$

$x=100$.

$100 P(100) - 1 = \frac{1}{99!} (99 \cdot 98 \dots 1) = \frac{1}{99!} \cdot 99! = 1$

$100 P(100) = 2$

$P(100) = \frac{2}{100} = \frac{1}{50}$



$x, y \in \mathbb{N}$

Trick

express as difference of 2 squares which equates to an integer

$2xy + x^2y^2 - 14xy + 49 = x^2 + y^2 + 2xy$

$x^2y^2 - 12xy + 49 = (x+y)^2$

$(xy^2 - 12xy + 36) + 13 = (x+y)^2$

$(xy-6)^2 + 13 = (x+y)^2$

$(x+y)^2 - (xy-6)^2 = 13 \Rightarrow (x+y+xy-6)(x+y-xy+6) = 13$

$(x+y-7)^2 = x^2 + y^2$

$x+y+xy-6 = 13$

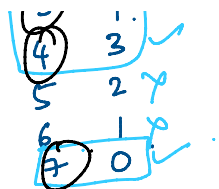
$x+y-xy+6 = 1$

$2(x+y) = 14$

$x+y = 7$

x	y
6	7
1	6
2	5
3	4
4	3
5	2

1×13



$$(x+y) - (xy-0) = 10$$

ix.13

$$(xy-7)^2 = x^2 + y^2$$

$$\text{Sum} = 0+3+4+7 = \underline{14}$$

Let $x = \sqrt{3-\sqrt{5}}$ and $y = \sqrt{3+\sqrt{5}}$. If the value of the expression $x - y + 2x^2y + 2xy^2 - x^4y + xy^4$ can be expressed in the form $\sqrt{p} + \sqrt{q}$ where $p, q \in \mathbb{N}$, then $(p+q)$ has the value equal to
 (A) 410 (B) 610 (C) 510 (D) 540

$$3 + \sqrt{5} = \frac{1}{2} (6 + 2\sqrt{5}) = \frac{1}{2} (\sqrt{5} + 1)^2 \quad y = \sqrt{3 + \sqrt{5}} = \frac{1}{\sqrt{2}} (\sqrt{5} + 1) = \frac{\sqrt{2}(\sqrt{5} + 1)}{2}$$

$$3 - \sqrt{5} = \frac{1}{2} (\sqrt{5} - 1)^2 \quad x = \frac{\sqrt{2}(\sqrt{5} - 1)}{2}$$

$$(x-y) + 2xy(x+y) - xy(x^3 - y^3) = (x-y) + 2xy(x+y) - xy[(x-y)^3 + 3xy(x-y)]$$

$$x+y = \frac{\sqrt{2}}{2} \cdot 2\sqrt{5} = \sqrt{10}$$

$$xy = \frac{2}{4} (5-1) = 2$$

$$x-y = \frac{\sqrt{2}}{2} (-2) = -\sqrt{2}$$

$$\begin{aligned} &= -\sqrt{2} + 4\sqrt{10} - 2[-2\sqrt{2} + 6(-\sqrt{2})] \\ &= -\sqrt{2} + 4\sqrt{10} + 4\sqrt{2} + 12\sqrt{2} = 15\sqrt{2} + 4\sqrt{10} \\ &= \sqrt{450} + \sqrt{160} = \sqrt{p} + \sqrt{q} \end{aligned}$$

$$\underline{p+q = 610}$$

Consider three natural numbers x, y and z such that when divided by 5, 7 and 11 respectively each leave remainder k . Then the expression $xyz - k(xy + yz + zx) + k^2(x + y + z) - k^3$ is necessarily divisible by
 (A) 35 (B) 77 (C) 55 (D) 385

More than 1 answer is correct.

$$x = 5a + k \quad y = 7b + k \quad z = 11c + k$$

$$x = 5a \quad y = 7b \quad z = 11c$$

$$xyz = 5 \times 7 \times 11 abc = 385 abc$$

$$0 \leq k \leq 4$$

k=0

Let $P(x)$ be a polynomial such that $x \cdot P(x-1) = (x-4)P(x) \forall x \in \mathbb{R}$. Then the degree of $P(x)$ cannot be

(A) 2 ✓ (B) 3 ✓ (C) 4 (D) 5 ✓

More than one answer

$$x P(x-1) = (x-4) P(x)$$

factors \rightarrow for what value of x

$x=0$

$$0 = -4 P(0) \Rightarrow \underline{P(0) = 0} \Rightarrow x \text{ is a factor of } P(x) \quad \underline{P(x) = 0}$$

$x=1$

$$1 \cdot \underline{P(0)} = -3 P(1) \quad 0 = -3 P(1) \Rightarrow \underline{P(1) = 0} \Rightarrow (x-1) \text{ is a factor.}$$

$x=2$

$$2 \cdot \underline{P(1)} = -2 P(2) \Rightarrow \underline{P(2) = 0} \Rightarrow (x-2) \text{ is a factor.}$$

$x=3$

$$3 \cdot \underline{P(2)} = -P(3) \Rightarrow \underline{P(3) = 0} \Rightarrow (x-3) \text{ is a factor.}$$

$x=4$

$$4 \cdot \underline{P(3)} = 0 \Rightarrow \underline{0 = 0}$$

$$P(x) = Ax(x-1)(x-2)(x-3)$$

\Downarrow
degree = 4

Find the first digit from left of 3^{62} where $\log 3 = 0.4771$

3

$$\log(3^{62}) = 62 \log 3 = 62 \times 0.4771 = \underline{29.5802}$$

$$\log 2 = 0.3010$$

$$\log 3 = 0.4771$$

$$\log 7 = 0.8451$$

$$\log 4 = 0.6020$$

$$\log 5 = \log\left(\frac{10}{2}\right)$$

$$= 1 - \log 2$$

$$= 1 - 0.301 = \underline{0.699}$$

$$\log 3^2 = 0$$

$$\text{No. of digits of } 8^4 = 1+1 = 2$$

$$\underline{0.9084} < 0.9562 = \log 9$$

$$0.5802 < 0.602 = \log 4$$

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More than 1 correct.

Let x, y, z be three positive real numbers such that
 $x + [y] + \{z\} = 13.2$
 $[x] + \{y\} + z = 14.3$
 $\{x\} + y + [z] = 15.1$
 where $[a]$ denotes the greatest integer $\leq a$ and $\{b\}$ denotes the fractional part of b , then
 (A) $xyz = 349.32$ (B) $x + y + z = 21.3$
 (C) $x + y - z = 4.9$ (D) $x - y + z = 27.6$

$$x = [x] + \{x\}$$

$$x = 2.3975$$

$$[x] = 2 \quad \{x\} = 0.3975$$

$$\begin{aligned} x + y + z &= 21.3 \\ - \{ [x] + \{y\} + z &= 14.3 \} \\ \hline \{x\} + [y] &= 7 \end{aligned}$$

$$\boxed{[y] = 7, \{y\} = 0}$$

$$\begin{aligned} x + [y] + \{z\} &= 13.2 \\ [x] + \{y\} + z &= 14.3 \\ \{x\} + y + [z] &= 15.1 \end{aligned}$$

$$2x + 2y + 2z = 42.6$$

$$\sim 11.17 = 21.3$$

$$\begin{aligned} x + y + z &= 21.3 \\ - [x + [y] + \{z\}] &= 13.2 \\ \hline \{y\} + [z] &= 8.1 \end{aligned}$$

$$\{y\} + [z] = 8.1$$

fr. Int

$$\boxed{[z] = 8, \{z\} = 0.1}$$

$$\begin{cases} x^2 + y^2 + z^2 = 0 \end{cases}$$

$$3.1 - 8.2 = 4.9$$

$$x = 6 \quad z = 8.2$$

$$y = 7.1$$

$$xyz = 349.32$$

$$2x + 4y + 6z = 70.2$$

$$x + y + z = 21.3$$

$$\rightarrow \{x\} + y + \{z\} = 15.1$$

$$\{x\} + \{z\} = 6.2$$

$$\{x\} = 6 \quad \{z\} = 0.2$$