24 November 2023 07:41 PM

4.23 Let A be the event of getting exactly 3 defectives in examination of 8 record players

and B → event that the 9th piece examined is a defective one.

8 → record play

3, 5

$$\frac{{}^{4}C_3 \times {}^{11}C_5}{{}^{15}C_8} = P(A)$$

$$P(B/A) = \frac{1}{7}$$

$$15 - 8 = 7$$

$$P(A \cap B) = P(B/A) \cdot P(A)$$

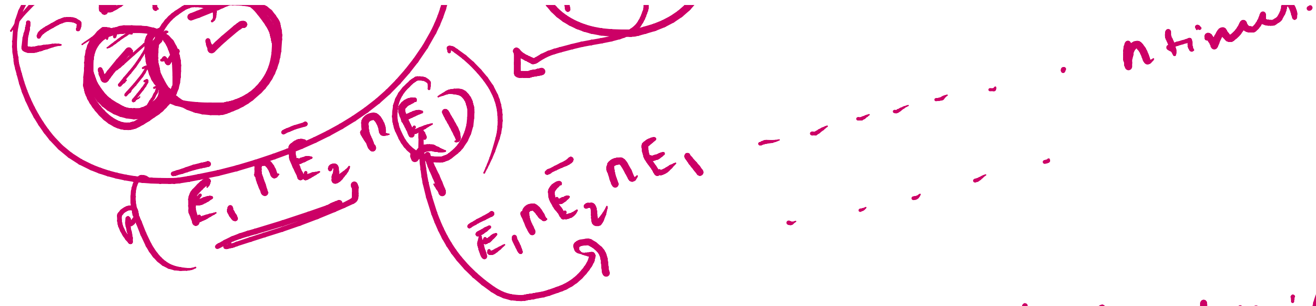
$E_1 \rightarrow$ A diamond $\qquad P(E_1) = \frac{13}{52} = \frac{1}{4}$

$E_2 \rightarrow$ B diamond $\qquad P(E_2) = \frac{13}{52} = \frac{1}{4}$

and $P(\bar{E}_1) = P(\bar{E}_2) = 3/4$

$P(E_1)$

. . . . n times.



Total probability theorem, Let 'p' denotes the probability that 'A' first wins.

$$P = P(E_1) + P(\bar{E}_1 \cap \bar{E}_2 \cap E_1) + P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_1 \cap \bar{E}_2 \cap E_1) + \cdots$$

By using compound probability theorem,

$$P = P(E_1) + P(\bar{E}_1) \cap P(\bar{E}_2) \cap P(E) + P(\bar{E}_1) \cdot P(\bar{E}_2) \cdot P(\bar{E}_1) \cdot P(\bar{E}_2) \cdot P(E_1)$$

$$P = \frac{1}{4} + \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} + \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} + \cdots$$

$$= \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^0 + \left(\frac{3}{4}\right)^2 \frac{1}{4} + \left(\frac{3}{4}\right)^4 \frac{1}{4} + \cdots$$

$$= \frac{1}{4}\left[\left(\frac{3}{4}\right)^0 + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^4 + \cdots\right]$$

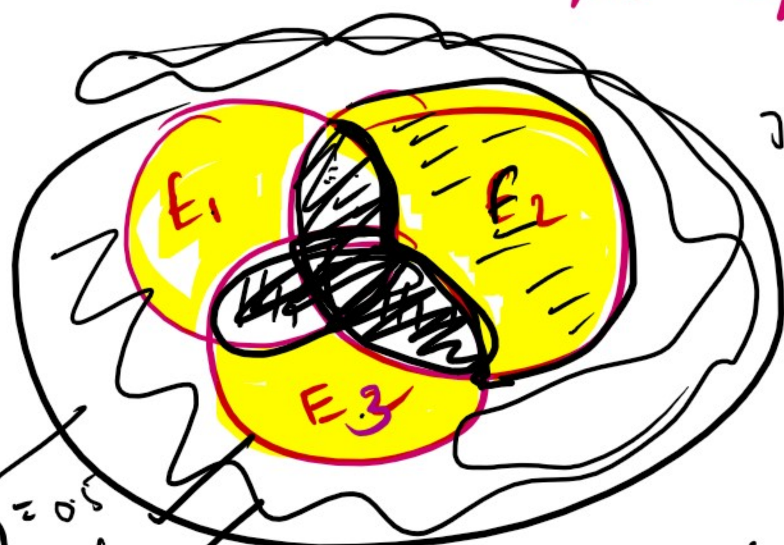
* Note = $\left[1 + x^2 + x^4 + \cdots = \frac{1}{1-x^2}\right]$

$$= \frac{1}{4}\left[\frac{1}{1-(3/4)^2}\right]$$

$$= \frac{1}{4}\left[\frac{1}{1-9/16}\right] = \frac{4}{7} \text{ (ans)}$$

$\therefore$ probability of B getting diamond first is $q = 1-p = 1-\frac{4}{7} = \frac{3}{7}$ (ans)

(b)



These three are mutually exclusive events:

(i) $\bar{E}_1 \cap \bar{E}_3 \cap E_2$

(ii) $\bar{E}_2 \cap \bar{E}_3 \cap E_1$

(iii) $\bar{E}_1 \cap \bar{E}_2 \cap E_3$

$P(E_1) = 0.5$
$P(E_2) = 0.6$
$P(E_3) = 0.8$

By additional probability theorem,

$$P = P(\bar{E}_1 \cap \bar{E}_3 \cap E_2) + P(\bar{E}_2 \cap \bar{E}_3 \cap E_1) + P(\bar{E}_1 \cap \bar{E}_2 \cap E_3)$$

$$= P(\bar{E}_1) \cdot P(\bar{E}_3) \cdot P(E_2) + P(\bar{E}_2) \cdot P(\bar{E}_3) \cdot P(E_1) + P(\bar{E}_1) \cdot P(\bar{E}_2) \cdot P(E_3)$$

$$= 0.26.$$

Ans 0.70.

*

E_4 $\quad A_1 \quad A_2 \quad A_3$

E_3 $\quad \bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3$

$P(E_1 \mid E)$

$E \rightarrow E_3 \cup E_4$

$A_1 \cap A_2 \cap \bar{A}_3$

$A_1 \cap A_3 \cap \bar{A}_2$

$E_3 \rightarrow$ No simp chng

$B, C, D \rightarrow E_4$

$$P(E_1 \mid E) = \left[\quad \right]$$

$$P(E) = P(E_3) + P(E_4)$$

all same

$B \quad C \quad D \rightarrow A_1 \quad A_2 \quad A_3$

$X \rightarrow \bar{A}_1 \quad \bar{A}_2 \quad \bar{A}_3$

$$P(E_3) = P(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3)$$

$$P(E_4) = P(A_1 \cap A_2 \cap \bar{A}_3) + P(A_1 \cap A_3 \cap \bar{A}_2)$$

$$P(\overline{E_4}) = P(A_1 \cap A_2 \cap A_3) + P(A_1 \cap A_3 \cdots$$
$$+ P(A_2 \cap A_3 \cap \overline{A_1})$$