19 February 2024 07:40 AM

POISSON DISTRIBUTION

pmf of discrete rv x with parameter λ is

$$P(X=x) \text{ or } f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad \text{for } x = 0, 1, 2, \ldots, \infty$$

$$= 0 \quad \text{otherwise.}$$

Q 10.14

$\Sigma f = N = 200$

$\Sigma f x = 122$

$\therefore$ mean, $\bar{x} = \frac{1}{N} \Sigma f x = \frac{122}{200} = 0.61$

mean of Poisson distribution $\Rightarrow \lambda = 0.61$

$\therefore$ Expected frequency of Poisson distribution is

$$f(x) = N \left(e^{-\lambda} \frac{\lambda^{x}}{x!} \right) \quad \text{for } x = 0, 1, 2, 3, 4$$

$$f(x) = 200\, e^{-0.61} \frac{(0.61)^{x}}{x!}$$

Here $e^{-0.61} = 0.5435$

x	$f(x)$
0	$200 \times 0.5435 \times \frac{(0.61)^0}{0!} = 108.7$
1	$200 \times 0.5435 \times \frac{(0.61)^1}{1!} = 108.7 \times 0.61 = 66.3$
2	$\vdots \quad \frac{(0.61)^2}{2!}$
3	$\vdots \quad \frac{(0.61)^3}{3!}$
	$(0.61)^4$

3	⋮	$3!$
4	⋮	$\frac{(0.61)^4}{4!}$

Conditions for approximation of BD to PD.

(i) $n \to \infty$ (large)
(ii) $p \to 0$
(iii) $np \to \lambda$

more specifically
$n \geq 20$
$p < 0.5$
then $np \to \lambda$.

then $P(x)$ or $f(x) = \frac{e^{-\lambda} \lambda^x}{x!}$

if we change 'λ' by 'np' then
limiting form of PD is $f(x) = \frac{e^{-(np)} (np)^x}{x!}$.

Ex 10.17:

$n = 100$

$p = \frac{1}{100} = 0.01 \qquad q = \frac{99}{100} = 0.99$

$f(x) = {}^nC_x \, p^x q^{n-x} = {}^{100}C_x (0.01)^x (0.99)^{100-x}$

for $x = 0, 1, 2, \ldots, 10$

BINOMIAL

x $f(x)$

0    $f(0) = {}^{100}C_0 (0.01)^0 (0.99)^{100} = (0.99)^{100} =$

1

2

3

4
5
⋮
10

$n = 100$ (large)
$p = 0.01$ (small)
$\therefore$ $np \to \lambda$ or $np = \lambda = 100 \times 0.01 = 1$

$$\therefore \quad p(x) \text{ or } f(x) = e^{-\lambda} \frac{\lambda^x}{x!}$$

$$= e^{-1} \frac{(1)^x}{x!}$$

$$f(x) = \frac{e^{-1}}{x!}$$

x	$f(x)$
0	$f(0) = e^{-1}/0! = e^{-1} = 0.366$
1	$f(1) = e^{-1}/1! = e^{-1} = 0.366$
2	$f(2) = e^{-1}/2! = e^{-1}/2 = 0.366/2 = 0.185$
⋮	$f(3) = e^{-1}/3! = e^{-1}/6 = 0.366/6 = 0.061$
10	⋮

32 :

$n = 10 \quad p = 1/50$

Here $f(x) = e^{-\lambda} \frac{\lambda^x}{x!}$

$np = \lambda = 10 \times \frac{1}{50} = 1/5 = 0.2$

$$f(x \le 2) = f(0) + f(1) + f(2)$$

$$= \frac{e^{-0.2} (0.2)^0}{0!} + \frac{e^{-0.2} (0.2)^1}{1!} + \frac{e^{-0.2} (0.2)^2}{2!}$$

In BD:

34 : mean is 6 sd is $\sqrt{2}$ n p, q ?

34: mean is 6 $\quad$ sd is $\sqrt{2}$ $\quad$ n, p, q ?

$\boxed{np = 6}$ $\quad$ $\sqrt{npq} = \sqrt{2}$

$npq = 2$

$6q = 2$

$\boxed{q = \frac{2}{6} = \frac{1}{3}}$

$p + q = 1$

$p = 1 - q$

$q = 1 - p$

$\therefore \boxed{p = 1 - q = 1 - 1/3 = 2/3}$

$np = 6$

$n \times \frac{2}{3} = 6$

$n = \cancel{6} \times \frac{3}{\cancel{2}}$

$\boxed{n = 9}$