07 August 2023 16:32

Question 1: If y is a real number, what is the difference in the maximum and minimum values obtained by $\frac{y+5}{y^2+5y+25}$?

$\frac{1}{5}$ $-\frac{1}{15}$

$\frac{1}{5}-\left(-\frac{1}{15}\right)$

$\frac{1}{5}+\frac{1}{15}=\frac{4}{15}$

a) 2/15

b) 4/15 ✓

c) 1/5

d) 1/15

$y=\frac{f(x)}{g(x)}$ → rational function.

$y+5$ → linear.

$y^2+5y+25$ → quadratic → dominant.

$$z=\frac{y+5}{y^2+5y+25}$$

$$zy^2+5zy+25z=y+5$$

$$zy^2+(5z-1)y+(25z-5)=0 \quad \text{quadratic in } y.$$

y is real ⇒ roots are real ⇒ $D\geq 0$

$$(5z-1)^2-4z(25z-5)\geq 0$$

$$25z^2-10z+1-100z^2+20z\geq 0$$

$$-75z^2+10z+1\geq 0$$

$$75z^2-10z-1\leq 0$$

$$75z^2-15z+5z-1\leq 0$$

$$15z(5z-1)+1(5z-1)\leq 0$$

$$(5z-1)(15z+1)\leq 0$$

$z\leq\frac{1}{5}$ & $z\geq-\frac{1}{15}$ ✓.

$z\geq\frac{1}{5}$ & $z\leq-\frac{1}{15}$ ✗

$ab\leq 0$: $a<0,\ b>0$ or $a>0,\ b<0$.

$$\underset{\text{min}}{-\frac{1}{15}}\leq z\leq\underset{\text{max}}{\frac{1}{5}}$$

find the maxima and minima of $\boxed{5\cos x+12\sin x}+4$

$$-\sqrt{a^2+b^2}\leq a\sin x+b\cos x\leq\sqrt{a^2+b^2}$$

$$4+(-13)\leq 5\cos x+12\sin x+4\leq 13+4$$

-9 $\qquad$ 17

22

$x=-8$

Question 2: What is the maximum value of g(x) = 22 – |x+8| and for what value of x is this reached?

$g(x) = 22 - |x+8|$ max

min $= 0$

$x+8=0$

$x=-8$

$y = a - x.$

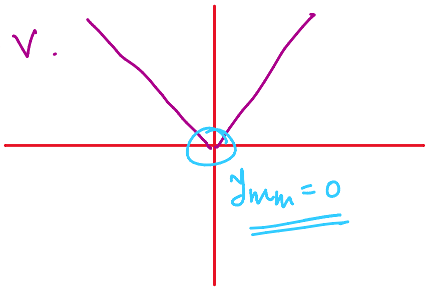
$x+y = a$

$y_{max} \rightarrow x_{min}$

$\boxed{y=|x|} \rightarrow V.$

$y = x,\ x \geq 0$

$= -x,\ x < 0$



Question 3: If $2 \leq x \leq 3$ and $4 \leq y \leq 5$, what are the minimum and maximum values of $\frac{y+x}{y+2x}$

a) 9/10, 7/8

b) 10/11, 8/9

c) 7/10, 7/9

d) None of these

min $= \frac{4+3}{4+6} = \frac{7}{10}.$

max $= \frac{5+2}{5+4} = \frac{7}{9}.$

$\frac{y+x}{y+2x} = \frac{\frac{y}{x}+1}{\frac{y}{x}+2} = \frac{\left(\frac{y}{x}+1\right)}{\left(\frac{y}{x}+1\right)+1} = \frac{1}{1+\frac{1}{\left(\frac{y}{x}\right)+1}}$

$\frac{a}{b}$ ↑max ↓min $\Rightarrow \left(\frac{a}{b}\right)_{max}$

$\left(\frac{y}{x}\right)_{max} \rightarrow \frac{y_{max}}{x_{min}} = \frac{5}{2}.$

$\left(\frac{y}{x}\right)_{max} \rightarrow \left(\frac{y}{x}+1\right)_{max} \rightarrow \frac{1}{\frac{y}{x}+1} \rightarrow$ min

$1+\frac{1}{\frac{y}{x}+1} \rightarrow$ min $\rightarrow \frac{1}{1+\frac{1}{\left(\frac{y}{x}\right)+1}}$ max

Question 4: If $x^2–5x+4 <= 0$ and $y^2 – 6y + 5 <= 0$, what are the maximum and minimum values of $\frac{y+7x}{y+4x}$

a) 29/17 , 12/9

b) 23/13, 11/8

c) 25/11, 14/11

d) None of these

$\frac{5+7}{5+4}$

$(y-5)(y-1) \leq 0 \Rightarrow 1 \leq y \leq 5$

$(x-4)(x-1) \leq 0 \Rightarrow 1 \leq x \leq 4$

$\frac{y+7x}{y+4x} = \frac{y+4x}{y+4x} + \frac{3x}{y+4x} = 1 + \frac{3x}{y+4x} = 1 + \frac{3}{\left(\frac{y}{x}\right)+4}$

$\frac{y_{max}}{x_{min}} \leftarrow \left(\frac{y}{x}\right)_{max} \rightarrow \left(\frac{y+7x}{y+4x}\right)_{min}$

Question 5: If X and Y are positive real numbers and 3X+4Y = 14, what is the maximum value of $X^3 * Y^4$?

a) 64

b) 128

c) 2187

d) None of these

AM ≥ GM.

$$\frac{x+x+x+y+y+y+y}{7} \geq (x^3y^4)^{1/7}.$$

$$\frac{14}{7} \geq (x^3y^4)^{1/7}.$$

$$2 \geq (x^3y^4)^{1/7}$$

$$2^7 \geq x^3y^4.$$

$x+y=10$ find $(xy)_{max}$

$$\frac{x+y}{2} \geq \sqrt{xy}.$$

$$\frac{10}{2} \geq \sqrt{xy}.$$

$$25 \geq xy$$

$a+b+c+d=30$, a, b, c, d are natural numbers.

find the minimum value of $(a-b)^2+(a-c)^2+(a-d)^2$

$[(a-b)^2]_{min} = ?$ 0

$0 + 1 + 1 = 2$

$a=b$

$a=b=c=d=\frac{30}{4}=7.5$

7, 7, 8, 8

$a=7$ $c=8$

$b=7$ $d=8$