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Probability

Some important terms:

(1) Random Experiment: is an act that can be repeated under similar conditions, whose result depend on chance and cannot be predicted in advance.

↓ Ex: tossing a coin

(2) Event: outcome of random experiment.

↳ event is getting (head or tail).

(3) Sample space: The set of all possible outcomes of a random experiment.

Ex: If a coin is tossed twice then the sample space is $S = \{(HH), (TT), (TH), (HT)\}$

(4) Mutually exclusive Events: when two or more events cannot occur simultaneously. That is joint occurance of two events are not possible.

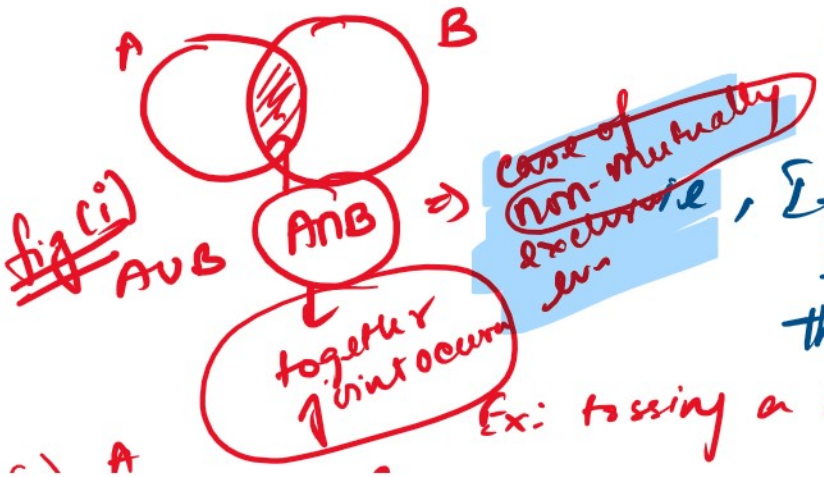

fig (i)

case of non-mutually exclusive ev-

If A and B are two events (mutually exclusive) then $P(A \cap B) = 0$

Ex: tossing a coin. Let A = head occurance of
Let B = ~~tail~~ occurance of tail

Ex: tossing a coin. Let A = head
Let B = tail
such that $P(A \cap B) = 0$

(ii) 

$A \cap B = \phi$.

Mutually exclusive

5. Exhaustive Events : Several events are exhaustive if at least one of them necessarily occurs whenever a random experiment is performed.

Ex: Again tossing a coin → you will get either head or tail.

CLASSICAL DEFINITION of PROBABILITY:

Suppose there are 'n' total number of elementary events in a sample space.
and n(E) is the no. of event favourable to event E.
Then probability of event E is defined as

$$P(E) = \frac{n(E)}{n}$$

i.e The classical defn of probability is defined as the ratio of the no. of favourable events to the no. of total elementary events.

Ex: What is the probability of getting odd

Ex: What is the probability of getting odd number when you roll a die?

Let E = event of getting an odd no.
$n(E)$ = no. of odd on the face of die = 3
n = total no. of outcomes = 6

$$\therefore P(E) = \frac{n(E)}{n} = \frac{3}{6} = \frac{1}{2} \text{ (ans)}$$

Properties: ① $0 \le P(E) \le 1$

if $P(E) = 0$ (0 chance of occurring)

if $P(E) = 1$ → sure event (100% chance of occurring)

$n(E)$ ⇒ tossed twice = 2
E } tossing
A ⇒ say head
$n(A)$ ⇒ no. of heads.

other approaches to probability theory:

(a) The frequency definition:

Suppose a random experiment E is repeated n times under same essential conditions and an event A, connected with E is found to occur $n(A)$ times.

The ratio $\frac{n(A)}{n}$ is the relative frequency

toss 3 times: $S = \{(HHH)\ (HHT)\ (HTH)\ (THH)\ (THT)\ (HTT)\ (TTH)\ (TTT)\}$

How many outcomes? 8

How many outcomes?

$2^n = 2^3 = 8$

$n \to \infty$ Large nos

The limiting value of $\frac{n(A)}{n}$ as n tends to infinity is called the probability of event A, denoted by $P(A) = \lim_{n \to \infty} \frac{n(A)}{n}$.

(3) The axiomatic approach :

A very sophisticated approach to the definition of probability is 'axiomatic approach".

Defn:

Let S be a sample space of random experiment E and A be any event connected with E i.e $A \subseteq S$. The probability of A is a number associated with A, to be denoted by P(A) such that the following axioms are satisfied:

(a) $P(A) \geq 0$

(b) If A be a sure event, then $P(A) = 1$.

Note:

Let there be two events A and B (non mutually exclusive) i.e joint occurance

mutually exclusive
ie joint occurance of A and B is possible.

Total probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

↓

Now if A and B are mutually exclusive events then $P(A \cap B) = 0$

then total probability is $P(A \cup B) = P(A) + P(B)$

(a) $P(A) \geq 0$

(b) If A be a sure event, then $P(A) = 1$.

(c) for any finite no. or countably infinite no. of mutually exclusive events $A_1, A_2 \ldots$ of S

$$P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \ldots$$

similarly if we have $A_1 A_2 \ldots A_n$ (i.e. n) no. of mutually exclusive events then

$$P(A_1 \cup A_2 \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$$

and if we have countably infinite mutually exclusive events like $A_1, A_2, A_3 \ldots$

(3) then $P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + \ldots$

Q1 When two unbiased coins are tossed, what is the probability of obtaining (a) 3 heads

(b) not more than 3 heads.

Sol $S = \{(HH), HT, TH, TT\}$

(a) Let A = no. of heads

$P(A) = 0$

(b) no. of heads not more than three

are 0, 1, 2, 3 i.e 4

$\therefore P(A) = 4/4 = 1.$

Q2 Two coins are tossed. Find the probability of getting both heads (or) both tails.

Q3. Two dice with faces marked 1, 2, 3, 4, 5, 6 are thrown simultaneously and the points on the dice are multiplied together. Find the probability of product 12.

Total no. of possible outcomes are $6^2 = 36$.

favourable outcome (3, 4) (4, 3) (2, 6) (6, 2)

i.e $n = 4$

$\therefore$ Prob of getting points product of which are 12 $= \frac{4}{36} = \frac{1}{9}$ (ans).

HW A bag contains 6 white and 4 black balls. One ball is drawn. What is the probability that it is white?