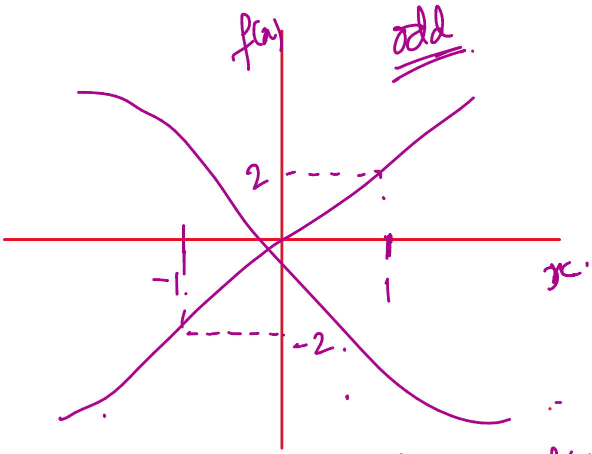
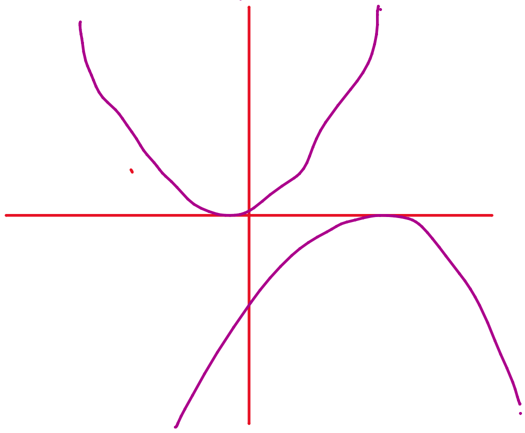
28 October 2023 17:01

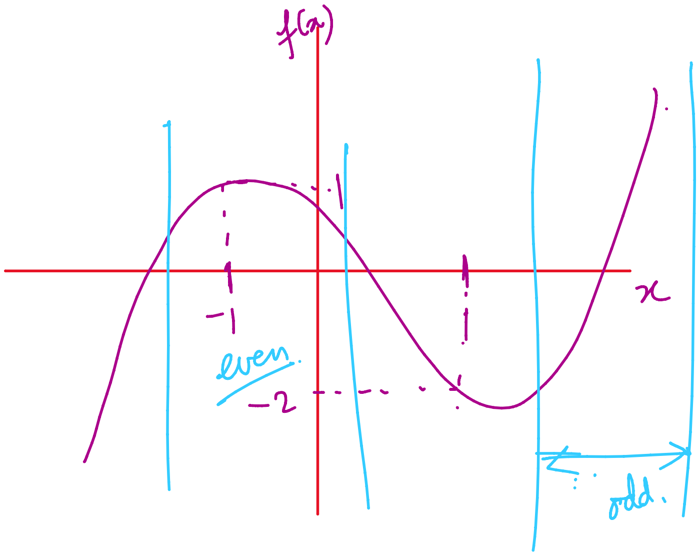
Odd and even functions



$f(-x) = -f(x)$

even $f(-x) = f(x)$





$f(-x) = -f(x)$ odd

$f(-x) = f(x)$ even

neither even nor odd

$$f(x) = \frac{x^2 + x + 2}{x^2 - 2x + 3}$$

Rational function

$$f(x) = \frac{g(x)}{h(x)}$$

find the range of $f(x)$

$$y = \frac{x^2+x+2}{x^2-2x+3}$$

$$yx^2 - 2yx + 3y = x^2 + x + 2$$

$$(y-1)x^2 - (2y+1)x + (3y-2) = 0$$

for x to be real $\quad D \geq 0$

$$(2y+1)^2 - 4(y-1)(3y-2) \geq 0.$$

$$4y^2+4y+1 - 4(3y^2-5y+2) \geq 0$$

$$4y^2-12y^2+4y+20y+1-8 \geq 0$$

$$-8y^2+24y-7 \geq 0$$

$$8y^2-24y+7 \leq 0. \rightarrow \text{roots are } \alpha \,\&\, \beta. \quad (\alpha > \beta)$$

$$(y-\alpha)(y-\beta) \leq 0.$$

$$\beta \leq y \leq \alpha$$

$ax^2+bx+c=0.$

$x = \dfrac{-b \pm \sqrt{D}}{2a}.$

$\boxed{8y^2-24y+7} = 0.$

$D = 24^2 - 4 \times 56.$

$= 576 - 224$

$= 352$

$$y = \frac{24 \pm \sqrt{352}}{16} = \frac{24 \pm \sqrt{16 \times 22}}{16} = \frac{24 \pm 4\sqrt{22}}{16} = \frac{6 \pm \sqrt{22}}{4}$$

$\alpha = \dfrac{6+\sqrt{22}}{4}$

$\beta = \dfrac{6-\sqrt{22}}{4}$

$$\boxed{\frac{6-\sqrt{22}}{4} \leq y \leq \frac{6+\sqrt{22}}{4}}$$

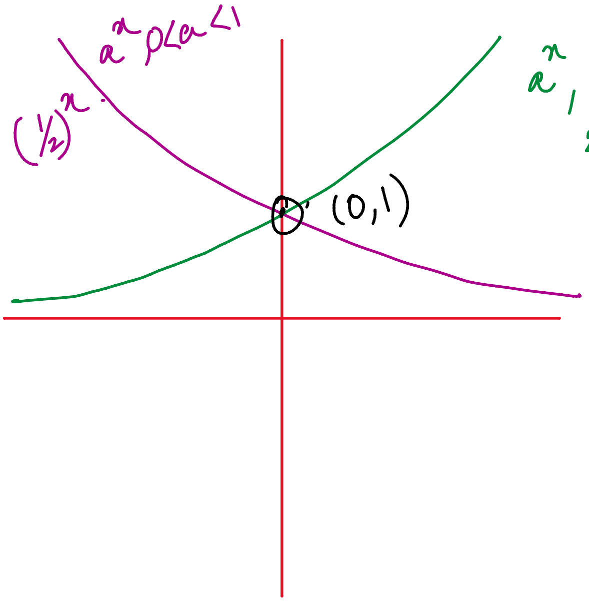
exponential functions

$f(x) = a^{f(x)}$

$f(x) = a^x \quad a > 0$

$0 < a < 1$: $\left(\frac{1}{2}\right)^x$

$a > 1$: $2^x.$



$a^x,\ a>1$

2^x

$a^x > 0$

Imp. $\quad y = f(x)^{g(x)}$

(x^2-2x+4) $\quad$ b .

$$\underbrace{(x^2+5x+5)}_{a}^{\overbrace{(x^2-2x+4)}^{b}} = 1 \qquad a^b = 1$$

- a = any real value except 0, $b = 0$
- $a = 1$, b = any real no.
- $a = -1$, b = even integer

Case 1 $b = 0$.

$x^2 - 2x + 4 = 0$

$D = 4 - 16 = -12$.

$\Rightarrow$ no roots.

Case 2

$x^2 + 5x + 5 = 1$

$x^2 + 5x + 4 = 0$

$(x+4)(x+1) = 0$

$\boxed{x = -4, -1}$

Case 3 $x^2 + 5x + 5 = -1$

$x^2 + 5x + 6 = 0$

$(x+2)(x+3) = 0$ $\boxed{x = -2, -3}$

x	x^2-2x+4
-2	12
-3	19 ✗

$\boxed{x = -1, -2, -4}$

$$f(x) = \frac{2x^2 + 3x + 4}{3x^2 + 2x - 1}$$

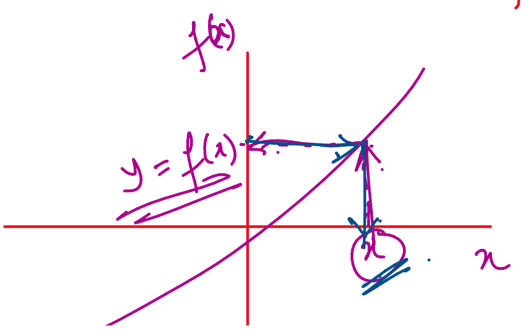
find the value of $f(x)$ when $x \to \infty$

$$f(x) = \frac{2 + \frac{3}{x} + \frac{4}{x^2}}{3 + \frac{2}{x} - \frac{1}{x^2}}$$

$x \to \infty$ $\quad \frac{1}{x} \to 0$

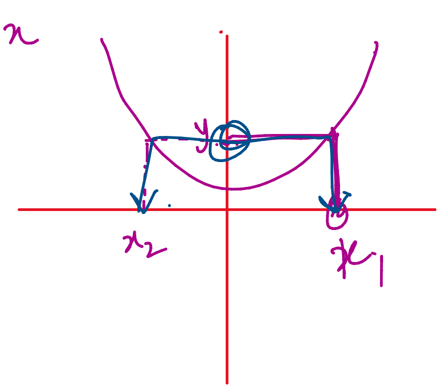
$\frac{2}{3}$

Inverse functions



$y = f(x)$ start from x and find $f(x)$

$f^{-1}(y) = x$ start from $f(x)$ and find x.



$f^{-1}(y)$ is not defined

If $f(x)$ is strictly increasing or strictly decreasing then $f^{-1}(x)$ exists $\Rightarrow$ $f'(x) > 0$ or $f'(x) < 0$

$$f(x) = \frac{x^2+x+2}{x^2-x+1}$$

$$f'(x) = \frac{(2x+1)(x^2-x+1) - (2x-1)(x^2+x+2)}{(x^2-x+1)^2}$$

$$= \frac{\cancel{2x^3} - 2x^2 + 2x + x^2 - x + 1 - (\cancel{2x^3} + 2x^2 + 4x - x^2 - x - 2)}{(x^2-x+1)^2}$$

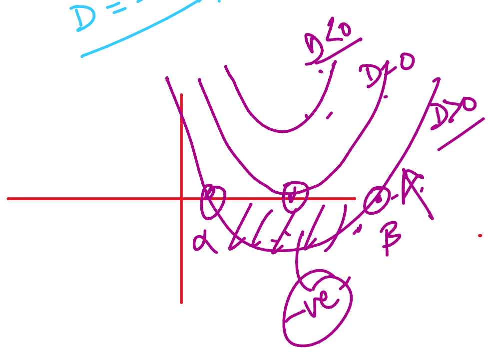
x^2-x+1

$D = -ve$

$(x-\alpha)(x-\beta)$

$$= \frac{-2x^2-2x+3}{(x^2-x+1)^2} = -\frac{(2x^2+2x-3)}{(x^2-x+1)^2}$$

+ve



$2x^2+2x-3$

$D = 4+24 > 0$

$$y = \frac{x+2}{x-1} \qquad y' = \frac{\cancel{x}-1-(\cancel{x}+2)}{(x-1)^2} = \frac{-3}{(x-1)^2}$$

$y' < 0$ except for $x = 1$

+ve except for $x=1$

swap x and y.

$$x = \frac{y+2}{y-1}$$

write y in terms of x.

$y-1$

write y in terms of x.

$xy - x = y + 2$

$y(x-1) = x+2$.

$\boxed{y = \frac{x+2}{x-1}}$

$f^{-1}(x)$.