

SLRM:

$$PRF: Y_i = \alpha + \beta X_i + u_i$$

$$SRE: \hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i, \text{ where } e_i = (Y_i - \hat{Y}_i)$$

Q.  $\hat{Y}$  and  $e$  are uncorrelated.

$$\text{corr}(\hat{Y}, e) = \frac{\text{cov}(\hat{Y}, e)}{\sqrt{\text{var}(\hat{Y})} \sqrt{\text{var}(e)}} = 0$$

$$\begin{aligned} \therefore \text{cov}(\hat{Y}, e) &= \frac{1}{n} \sum (\hat{Y}_i - \bar{\hat{Y}})(e_i - \bar{e}) \\ &= \frac{1}{n} \sum (\hat{Y}_i - \bar{\hat{Y}}) e_i \quad [\because \bar{e} = 0 \text{ from } NE(i)] \\ &= \frac{1}{n} \sum \hat{Y}_i e_i - \frac{1}{n} \cdot \bar{\hat{Y}} (\sum e_i) \\ &= \frac{1}{n} \sum \hat{Y}_i e_i \quad \text{"0"} \\ &= \frac{1}{n} \sum (\hat{\alpha} + \hat{\beta} X_i) e_i \\ &= \frac{1}{n} \left[ \hat{\alpha} (\sum e_i) + \hat{\beta} (\sum e_i X_i) \right] \quad \text{"0"} \quad \text{"0"} = 0 \end{aligned}$$