

Probability Questions

Q1. In 10 independent throws of a defective dice, the probability that an even number will appear 5 times is twice the probability that an even number will appear 4 times.
 Find the probability that an even number will not appear at all in 10 independent throws of the dice.

Soln: $p \rightarrow$ probability of getting even no.
 $q = 1-p \rightarrow$ " " " not " " "

for x no. of success in 10 throws pmf of binomial distribution is $f(x) = {}^{10}C_x p^x q^{10-x}$

Acc to qn: $f(5) = 2f(4)$

$${}^{10}C_5 p^5 q^5 = 2 \times {}^{10}C_4 p^4 q^6$$

$$\frac{10!}{5!5!} p = 2 \times \frac{10!}{4!6!} q$$

$$p = \frac{5!5!}{4!6!} q \times 2$$

$$p = \frac{5 \times 4! \times 5!}{4! \times 6 \times 5!} q \times 2$$

$$p = \frac{5 \times 2 q}{3 \times 2}$$

$$P = \frac{5 \times 10^9}{3^8}$$

$$3P = 5^2$$

$$3P = 5(1-P) \Rightarrow 3P + 5P = 5$$

$$\text{or, } 8P = 5$$

$$\therefore P = \frac{5}{8}$$

\therefore probability of not getting an even no. at all,

$$f(x=0) = {}^{10}C_0 \left(\frac{5}{8}\right)^0 \left(\frac{3}{8}\right)^{10}$$

$$= \left(\frac{3}{8}\right)^{10} \text{ (ans)}$$

Q2. The overall % of failures in a certain examination is 40. What is the probability that out of 6 candidates at least 4 passed the exam?

Q3 10 cups tea are prepared $\left\{ \begin{array}{l} 5 \text{ in one way} \\ 5 \text{ in another way} \end{array} \right.$

Find probability that she would judge correctly all the cups, it being known to her that 5 are of each kind.

$$\frac{{}^5C_5}{{}^5C_5} \text{ or } \frac{{}^5C_5}{{}^5C_5}$$

$$(10) \Rightarrow 5 \text{ ways. } {}^{10}C_5 \text{ ways}$$

$\frac{10}{5}$ ways. Total ways in front of lady

${}^{10}C_5$ ways in front of lady

$$= \frac{10!}{5!5!} = \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1}$$

$(10) \rightarrow 5 \text{ ways} \cdot {}^{10}C_5 \text{ ways}$

$= 252 \rightarrow$ there are 252 ways of presenting the cups to lady.

for example: M M T T M T M M T M
 \therefore required probability is $\frac{1}{252}$

(*) When the cups are presented in 5 pairs i.e. total number presented $\rightarrow 2^5 = 32$

pairs possible: (M, T) or (T, M)



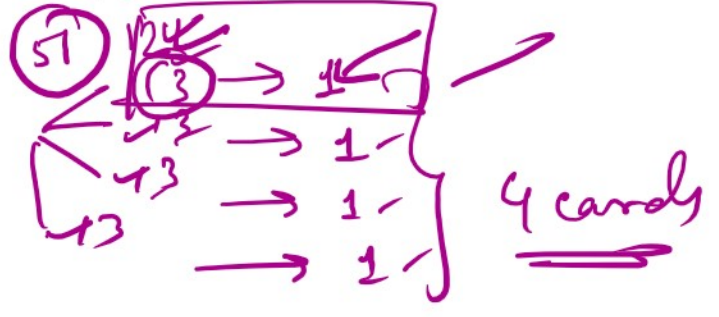
Required probability is $\frac{1}{32}$ (ans)

Q4

Four cards are drawn at random from a full pack. What is the probability that they belong to (i) 4 different suits. (ii) different suits and denominations.

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pack \Rightarrow 52 cards.



Total cases:

- first card \Rightarrow 52
- second draw \Rightarrow 51
- 3rd \Rightarrow 50
- 4th \Rightarrow 49

Total \Rightarrow $52 \times 51 \times 50 \times 49$

$n_y = 52 C_4$

- favourable cases:
- 1 card $\rightarrow 52$ } $13 C_1$
 - 2 card $\rightarrow 39$ } $13 C_1$
 - 3 card $\rightarrow 26$ } $13 C_1$
 - 4 card $\rightarrow 13$ } $13 C_1$

~~$52 \times 39 \times 26 \times 13$~~

\therefore Required probability = $\frac{\text{fav event}}{\text{Total event}}$

$\frac{13 C_1 \cdot 13 C_1 \cdot 13 C_1 \cdot 13 C_1}{52 C_4} = \frac{52 \times 39 \times 26 \times 13}{52 \times 51 \times 50 \times 49}$

\Rightarrow $\frac{1}{52} \times \frac{36}{51} \times \frac{22}{49} \times \frac{10}{49}$

□

