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Probability Questions

Q1. In 10 independent throws of a defective dice, the probability that an even number will appear 5 times is twice the probability that an even number will appear 4 times.
Find the probability that an even number will not appear at all in 10 independent throws of the dice.

Soln: $p \rightarrow$ probability of getting even no.
$q = 1-p \rightarrow$ " " not " " ".

for x no. of success in 10 throws pmf of binomial distribion is $f(x) = {}^{10}C_x p^x q^{10-x}$

Acc to qus: $f(5) = 2f(4)$

$${}^{10}C_5 p^5 q^5 = 2 \times {}^{10}C_4 p^4 q^6$$

$$\frac{\cancel{10!}}{5!\,5!} p = 2 \times \frac{\cancel{10!}}{4!\,6!} q$$

$$p = \frac{5!\,5!}{4!\,6!} q \times 2$$

$$p = \frac{5 \times \cancel{4!} \times \cancel{5!}}{\cancel{4!} \times 6 \times \cancel{5!}} q \times 2$$

$$p = \frac{5\,\cancel{10}\,q}{3\,\cancel{6}}$$

$$p = \frac{5 \times q \times 2}{3 \times 6}$$

$$3p = 5q$$

$3p = 5(1-p) \Rightarrow 3p + 5p = 5$

or, $8p = 5$

or, $\boxed{p = 5/8}$

$\therefore \boxed{q = 3/8}$

$\therefore$ probability of not getting an even no. at all,

$$f(x=0) = {}^{10}C_0 (5/8)^0 (3/8)^{10}$$

$$= \left(\frac{3}{8}\right)^{10} \text{ (ans)}$$

Q2. The overall % of failures in a certain examination is 40. What is the probability that out of 6 candidates at least 4 passed the exam?

Q3 10 cups tea are prepared — 5 in one way, 5 in another way.

Find probability that she would judge correctly all the cups, it being known to her that 5 are of each kind.

$\left(\frac{{}^{}C^T}{M}\right)$ or $\left(\frac{C^m}{T}\right)$

${}^{10}C_5$ ways in front of lady — total ways

(10) ⇒ 5 ways. ${}^{10}C_5$ ways Total

$^{10}C_5$ ways in front of lady

(10) $\Rightarrow$ 5 ways. $^{10}C_5$ ways

$$= \frac{10!}{5!\,5!} = \frac{\cancel{10}^{2}\times 9\times \cancel{8}^{2}\times 7\times \cancel{6}\times \cancel{5!}}{\cancel{5!}\ \cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times 1}$$

$$2\times 9\times 2\times 7$$

$= 252 \Rightarrow$ there are 252 ways of presenting the cups to lady.

for example: MMTTMTMMTM $\rightarrow$ 252 ways

$\therefore$ required probability is $\frac{1}{252}$.

(A) When the cups are presented in 5 pairs i.e total number presented $\Rightarrow 2^5 = 32$.

pairs possible: (M, T) or (T, M)

Required probability is $\frac{1}{32}$ (ans)

Q4 Four cards are drawn at random from a full pack. What is the probability that they belong to (i) 4 different suits.

(ii) different suits and denominations.

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pack ⇒ 52 cards.

51, 13 → 1

13 → 1

13 → 1

13 → 1

} 4 cards

Total cases:

first card drawn ⇒ 52

second draw ⇒ 51

3rd ⇒ 50

4th ⇒ 49

Total ⇒ $52 \times 51 \times 50 \times 49$

$^nC_4 = {}^{52}C_4$

favourable cases:

1 card → 52 $^{13}C_1$

2 card → 39 $^{13}C_1$

3 card → 26 $^{13}C_1$

4 card → 13 $^{13}C_1$

$52 \times 39 \times 26 \times 13$

$\therefore$ Required probability $= \dfrac{\text{Fav event}}{\text{Total event}}$

$$= \frac{52 \times 39 \times 26 \times 13}{52 \times 51 \times 50 \times 49} = \frac{\quad}{\quad}$$

$$\frac{^{13}C_1 \cdot {}^{13}C_1 \cdot {}^{13}C_1 \cdot {}^{13}C_1}{^{52}C_4}$$

1   2   3   4

$$\Rightarrow \frac{52}{52} \times \frac{36}{51} \times \frac{22}{50} \times \frac{10}{49}$$

$$\frac{52}{52} \times \frac{36}{51} \times \frac{22}{50} \times \frac{10}{49}$$

(ans)