18 February 2024 11:06 AM

Random Variables → probability distribution

Random Variables: Discrete (Probability mass function (pmf)), Continuous (probability density function (p.d.f)).

whole no. or finite ← Discrete

$x = 0, 1, 2, 3, \ldots 20$

$P = 0.1\ 0.1 \ldots 0.1$

$\boxed{\Sigma P = 1}$

Continuous → infinite, fractional/decimal → probability density function (p.d.f).

p.m.f

x	→ $P(x = x)$ or $f(x)$
0	0.3
1	0.3
2	0.4
	$\Sigma P = 1.0$

① X is a discrete r.v? or X follow p.m.f.?

Conditions : (a) for all values of x, $P(x = x)$ or $f(x) \geq 0$

(b) $\sum_x f(x) = 1$.

② X is a cont. r.v? or X follow p.d.f?

conditions: (a) for all values of x, $P(x = x)$ or $f(x) \geq 0$

(b) $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

Mean or Expectation of a r.v x.

① X is a discrete r.v.

$E(x) = \sum_x x \cdot P$

$V(x) = E(x^2) - [E(x)]^2$

$E(x^2) = \Sigma x^2 \cdot P$

$\bar{x} = \frac{1}{n} \Sigma x$

$= E(x) =$ mean

$Var(x) = \frac{1}{n} \Sigma x^2 - \bar{x}^2$

$E(x^2) - [E(x)]^2$

$\rightarrow E(x^2) = \sum x^2 \cdot p$

$E(x^2) - \{E(x)\}$

② x is a continuous r.v.

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

again $E(x^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)\, dx$

$$V(x) = E(x^2) - \{E(x)\}^2$$

Q: Calculate expected value of discrete r.v. x.

x	$P(x)$	$x \cdot p$
0	0.12	0
1	0.20	0.20
2	0.25	0.50
3	0.30	0.90
4	0.13	0.52

$\sum xp =$ ans.

$= 2.12$

$E(x) = \sum_x x \cdot P(x)$

$\therefore E(x) = 2.12$ (ans)

Binomial Distribution

① n no. of trials

② x is no. of success out of n trials.

③ p is the probab of success

④ q = prob of failure = 1-p

because p+q = 1

⑤ The probability mass function (p.m.f) of

5) The probability mass function (p.m.f) of a discrete r.v X that follows a binomial distribution with parameter n and p is written as

$$f(x) = {}^nC_x \, p^x \, q^{n-x} \quad ; x = 0, 1, \ldots, n$$
$$= 0 \quad ; \text{otherwise}$$

$${}^nC_x = \frac{n!}{x!(n-x)!}$$

$n! = n(n-1)(n-2)\cdots 2 \cdot 1.$

$5! = 5 \times 4 \times 3 \times 2 \times 1$

$10! = 10 \times 9 \times 8 \times \cdots 3 \times 2 \times 1.$

$${}^{10}C_5 = \frac{10!}{5!(10-5)!} = \frac{10!}{5!5!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times \cancel{5!}}{5! \, \cancel{5!}} = \frac{\cancel{10}^{2} \times \cancel{9}^{3} \times \cancel{8}^{4} \times 7 \times 6}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$$
$= 2 \times 3 \times 7 \times 6$
$= 6 \times 6 \times 7$
$= 6 \times 42$
$= 252$ (ans)

$${}^8C_3 = \frac{8!}{3!5!} = \frac{8 \times 7 \times \cancel{6} \times \cancel{5!}}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{5!}} = 56 \text{ (ans)}$$

Properties of BD:

1) Mean or Expectation
$E(x) = n \cdot p$

2) Variance $v(x) = n \cdot p \cdot q$
$= np(1-p)$

$\therefore SD(x) = \sqrt{npq}.$

Q A coin is thrown 7 times. Find the probability of getting at least 5 heads.

$n = 7 \quad P(x \geq 5)$ i.e $x = 5, 6, 7$

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$x = 0, 1, 2, 3, \ldots, 7$

p = prob of getting head $= 1/2$

$\therefore q = 1/2$

$$p(x \geq 5) = p(x=5) + p(x=6) + p(x=7)$$

$$= {}^7C_5 p^5 q^{7-5} + {}^7C_6 p^6 q^{7-6} + {}^7C_7 p^7 q^0$$

$$= \frac{7!}{5!2!}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^2 + \frac{7!}{6!1!}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^1 + 1\left(\frac{1}{2}\right)^7$$

$$= \frac{7 \times \cancel{6}^3}{\cancel{2} \times 1}\left(\frac{1}{2}\right)^7 + 7\left(\frac{1}{2}\right)^7 + \left(\frac{1}{2}\right)^7$$

$$\frac{7!}{6!} = \frac{7 \times \cancel{6!}}{\cancel{6!}} = 7$$

$$= \left(\frac{1}{2}\right)^7 [21 + 7 + 1]$$

$$= \left(\frac{1}{2}\right)^7 \times 29$$

$$= \frac{29}{128} \text{ (ans)}$$

$\boxed{0, 1, 2, \ldots 8}$ 1

(2) mean = 4 variance = 2

B.D: $np = 4$ $\quad npq = 2$

$4 \cdot q = 2$

$\therefore n \frac{1}{2} = 4$

$\boxed{n = 8}$

$\boxed{q = \frac{2}{4} = \frac{1}{2}}$

$\boxed{p = 1 - q = \frac{1}{2}}$

$$f(x) = {}^8C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{8-x}$$

for $x = 0, 1, 2, \ldots, 8$.

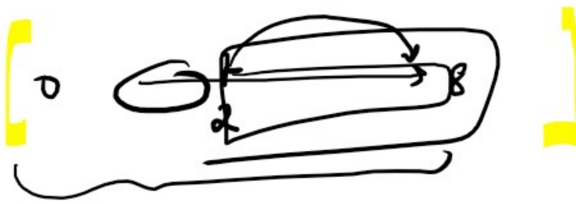
(i) at least 2 sum

$$P(x \geq 2) = 1 - P(x < 2)$$

$$= 1 - [P(x=0) + P(x=1)]$$

$$= 1 - \left[{}^8C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^8 + 8 \ldots \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^7\right]$$

[0 ... 8]



total = 1

$$\left[{}^8C_0 \left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right) + {}^8C_1 \left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)^7\right]$$

$$= 1 - \left[1\left(\tfrac{1}{2}\right)^8 + \frac{8!}{7!}\left(\tfrac{1}{2}\right)^8\right]$$

$$= 1 - \left[\left(\tfrac{1}{2}\right)^8 + 8\left(\tfrac{1}{2}\right)^8\right]$$

$$= 1 - \left(\tfrac{1}{2}\right)^8 (9)$$

$$= 1 - \frac{9}{256}$$

(ii) $P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$

$$= {}^8C_0\left(\tfrac{1}{2}\right)^0\left(\tfrac{1}{2}\right)^8 + {}^8C_1\left(\tfrac{1}{2}\right)^1\left(\tfrac{1}{2}\right)^7 + {}^8C_2\left(\tfrac{1}{2}\right)^2\left(\tfrac{1}{2}\right)^6$$ (ans)

$$= \frac{9}{256} + \boxed{\frac{8!}{6!\,2!}}\left(\tfrac{1}{2}\right)^8$$

$$= \frac{9}{256} + \frac{\cancel{8}^{4} \times 7 \times \cancel{6!}}{\cancel{6!}\,\cancel{2} \times 1}\left(\tfrac{1}{2}\right)^8$$

$$= \frac{9}{256} + \frac{28}{256} = \frac{37}{256} = 0.144$$

expected binomial frequencies

$\boxed{f(x) = \underbrace{N}_{150} \times \underbrace{{}^nC_x\, p^x\, q^{n-x}}}$

x	$f(x) = 150\,{}^nC_x\, p^x q^{n-x}$
0	
1	
2	
3	

$\bar{x} = \frac{1}{N}\Sigma f x = \frac{200}{150} = np$

$= $ [illegible]

N ... $\frac{}{150}$

$= \frac{\cancel{200}^{8}}{\cancel{4} \times \cancel{150}_{3}} = p$

$\Rightarrow \boxed{p = 1/3}$ $\quad \boxed{q = 1 - 1/3 = 2/3}$

3
4

$\boxed{\sigma^2 = \frac{1}{N}\sum x_i^2 f - \bar{x}^2}$

#

x	f	xf	$x_i^2 f$
0	4	0	0
1	20	20	20
2	40	80	160
3	40	120	360
4	20	80	320
5	4	20	100
	$\sum f = 128$	$\sum xf = 320$	$\sum x_i^2 f = 960$

$\bar{X} = \frac{1}{\sum f}\sum xf = \frac{320}{128} = 2.5$ $\Rightarrow np = 2.5$ or $p = \frac{2.5}{5} = 0.5$

$\sigma^2 = \frac{1}{N}\sum x_i^2 f - \bar{x}^2 = \frac{960}{128} - (2.5)^2$

$= 7.5 - 6.25$

$= 1.25$

$\therefore \sigma = \sqrt{1.25} = 1.118$

$\sigma^2 = 1.25 = npq$ $\qquad n = 5$

$\Rightarrow q = \frac{1.25}{2.5} = 0.5$ $\qquad p = 1/2$

$q = 1 - p = 1 - 0.5 = 0.5$ $\qquad q = 1/2$

Q $n = 8$ $p = 1/6 + 1/6 = 2/6 = 1/3$

and $q = 1 - p = 2/3$

$$\therefore P(x=3) = {}^nC_3 \; p^3 \; q^{n-3}$$

$$= {}^8C_3 \left(\frac{1}{3}\right)^3 \left(2/3\right)^{8-3}$$

$$= \frac{8!}{5!\,3!} \; \frac{1}{27} \times \frac{2^5}{3^5}$$

$$= \frac{8 \times 7 \times \cancel{6}^{3} \times \cancel{5!}}{\cancel{5!}\; \cancel{3} \times \cancel{2} \times 1} \; \frac{32}{27 \times 243}$$

$$= \frac{56 \times 32}{27 \times 243} = \frac{1792}{6561} = 0.27$$

Q $P(x=0) = {}^nC_0 \; p^0 \; q^{n-0}$

$$= {}^4C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^{4-0}$$

$$= 1 \cdot 1 \left(\frac{1}{3}\right)^4 = \frac{1}{81}$$ ✓

$$P(x=1) = {}^nC_1 \; p^1 \; q^{n-1}$$

$$= {}^4C_1 \left(\frac{2}{3}\right)^1 \left(1/3\right)^3$$

$$= 4 \cdot \; 2 \; (\;1\;)^4$$

$$= \frac{4!}{3!} \, 2 \left(\frac{1}{3}\right)^4$$

$$= 8 \times \frac{1}{81} = \frac{8}{81}$$

POISSON DISTRIBUTION:

A discrete rv x with parameter 'λ' is said to follow a probability mass fn (p.m.f) written as

$$P(X=x) \text{ or } f(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}, \quad \text{where } x = 0,1,2,\ldots\infty$$

$$= 0, \quad \text{elsewhere}$$

mean or expected value of x, $\boxed{E(x) = \lambda}$

variance, $V(x) = \lambda$

$SD(x) = \sqrt{\lambda}$

when $n \to \infty$, $p \to 0$ then $np \to \lambda$

Poisson Distribution.

Q12

n is large

$p = 0.01 \quad n = 100$

$$f(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$\lambda = np = 100 \times 0.01 = \cancel{100} \times \frac{1}{\cancel{100}} = 1.$$

$$\therefore f(x=2) = \frac{e^{-\lambda} \lambda^{2}}{2!} = \frac{e^{-1} (1)^2}{2!} = \frac{e^{-1} \cdot 1}{2}$$

$$= e^{-1}/2$$

$$= 0.367$$

(i) upto 2 or at most 2

(1) upto 2 or at most 2

$P(x \leq 2) = x = 0, 1, 2$

(2) less than 2

$P(x < 2) = x = 0, 1$

(3) at least 2

$P(x \geq 2) = 1 - P(x < 2) = 1 - [P(x=0) + P(x=1)]$

(4) more than 2

$P(x > 2) = 1 - P(x \leq 2) = 1 - [P(x=0) + P(x=1) + P(x=2)]$

$= \frac{0.367}{2}$

$= 0.183$

(ans).