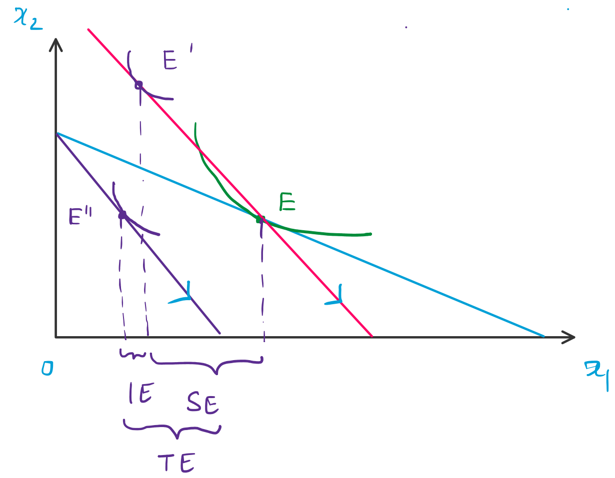
Q: $u(x_1, x_2) = x_1^2 x_2^3$. $M = 1000$, $P_2 = 20$. Suppose Price of Good 1 increases from 4 to 5. Decompose the total effect on Good 1. Show it graphically as well.

$\therefore$ $E \to E' = SE$

$E' \to E'' = IE$

$TE = SE + IE$.



Classification of Goods Based on Slutsky Decomposition:

Suppose, the price of Good 1 decreases. Decompose the total price effect. [For Good 1]

Case I: $SE < 0$, $IE > 0$ [Pt A]

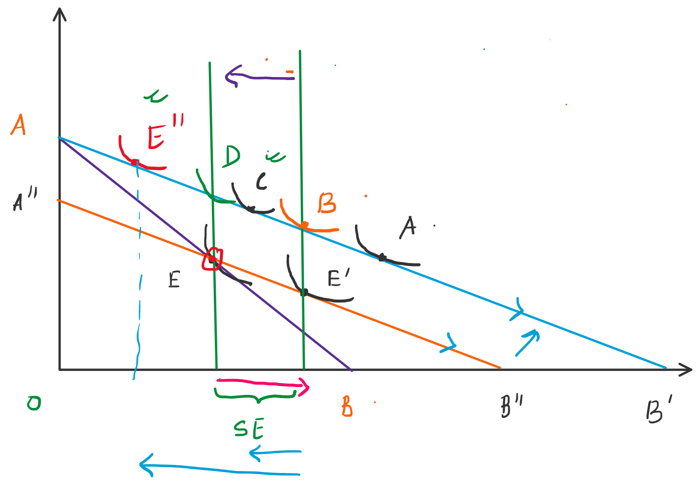
Normal Goods.

Case II: $SE < 0$, $IE = 0$ [Pt B]

Necessary Goods.

(*) **Case III:** $SE < 0$, $IE < 0$ [Pt C]

Inferior Goods.



Case IV: $SE < 0$, $IE < 0$, $|SE| = |IE|$ [Pt D]

No change in demand.

(*) **Case V:** $SE < 0$, $IE < 0$; $\underbrace{|IE| > |SE|}$ [Pt E'']

Giffen Good.

"All Giffen Goods are Inferior Goods but all Inferior Goods are not Giffen Goods"

Inferior Goods ⇒ obeys the Law of Demand ⇒ $|IE| < |SE|$
Giffen Goods ⇒ violates the Law of Demand ⇒ $|IE| > |SE|$

Hicksian Substitution Effect:-

Recall: For Slutsky SE: hold the money income constant [Graphically, push the B·L Illy till it passes through the old consumption bundle] ⇒ For Slutsky SE, hold the nominal income.

Utility ⇒ Real Income of the individual.

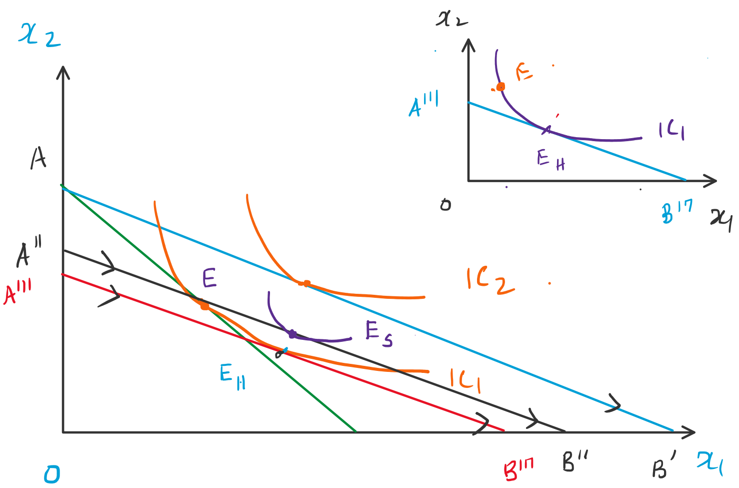
For Hicksian SE, utility is held constant. So, graphically push the new B·L ‖ly till it passes through the initial IC.

$A''B''$: Slutsky SE B·L.

$A'''B'''$: Hicksian SE B·L.

E_S: Equillibrium under Slutsky SE.

E_H: Equillibrium under Hicksian SE.



Q. Show that in a 2-Good Framework, both the goods cannot be simultaneously superior. [luxurious].

Inferior: $\frac{\partial x^*}{\partial M} < 0 \Rightarrow \left(\frac{M}{x} \cdot \frac{\partial x^*}{\partial M}\right) < 0 \Rightarrow \boxed{e_M < 0}$

Normal: $\frac{\partial x^*}{\partial M} > 0 \Rightarrow \left(\frac{M}{x} \cdot \frac{\partial x^*}{\partial M}\right) > 0 \Rightarrow e_M > 0$.

↳ Superior Goods $e_M > 1$.

$$e_M = \frac{\partial x^*/x^*}{\partial M^*/M} = \frac{\% \Delta x^*}{\% \Delta M}.$$

Max. $u = u(x_1, x_2)$ s.t $M = P_1x_1 + P_2x_2$

$\partial M/M \qquad \% \Delta M$

Max. $u = u(x_1, x_2)$ s.t $M = P_1 x_1 + P_2 x_2$.

For checking superiority, evaluate change in demand due to change in M.

B.L $\quad M = P_1 x_1 + P_2 x_2$.

Diff w.r.t M: $\quad 1 = P_1 \cdot \dfrac{\partial x_1}{\partial M} + P_2 \cdot \dfrac{\partial x_2}{\partial M}$.

$$1 = \frac{P_1 \cdot x_1}{M}\left(\frac{M}{x_1} \cdot \frac{\partial x_1}{\partial M}\right) + \frac{P_2 x_2}{M}\left(\frac{M}{x_2} \cdot \frac{\partial x_2}{\partial M}\right)$$

$$1 = \frac{P_1 x_1}{M}(e_{1M}) + \frac{P_2 x_2}{M} \cdot (e_{2M})$$

$\frac{P_1 x_1}{M}$ ↳ exp share on Good 1 (θ_1)

$\frac{P_2 x_2}{M}$ ↳ exp share on Good 2 (θ_2)

$$1 = \theta_1 e_{1M} + \theta_2 e_{2M}.$$

$$\theta_1 + \theta_2 = \frac{P_1 x_1}{M} + \frac{P_2 x_2}{M} = 1.$$

$$\theta_1 + \theta_2 = \theta_1 e_{1M} + \theta_2 e_{2M}.$$

$$\boxed{\underbrace{\theta_2}_{>0} \underbrace{(1 - e_{2M})} = \underbrace{\theta_1}_{>0} \underbrace{(e_{1M} - 1)}_{>0}}$$

$\theta_1, \theta_2 > 0$; Eg: Good 1 is superior $\Rightarrow e_{1M} > 1$.

$1 - e_{2M} > 0 \Rightarrow \boxed{e_{2M} < 1}$

$\therefore$ If $e_{1M} > 1$, we must have $e_{2M} < 1$.

$\therefore$ 2 Goods cannot be simultaneously superior.