

LINEAR ALGEBRA SESSION FOR 09.11.23

Linear Algebra Session

WTC: find a pattern.

30. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then $A^n =$

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 50 & 0 & 0 \\ 50 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & 0 & 0 \\ 48 & 0 & 0 \\ 48 & 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 4 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 24 & 1 & 0 \\ 24 & 0 & 1 \end{bmatrix}$ (1 mark)

2 4

2, 4, 8

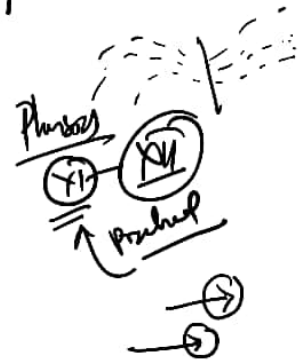
$$A^8 = \left| \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 2 & 1 & 0 \\ 2 & 0 & 1 & 2 & 0 & 1 \end{array} \right| = \left| \begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & 1 & 0 \\ 4 & 1 & 0 & 4 & 0 & 1 \\ 4 & 0 & 1 & 4 & 0 & 1 \end{array} \right|$$

$$A^{30} = \left| \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 45 & 1 & 0 & 45 & 1 & 0 \\ 45 & 0 & 1 & 45 & 0 & 1 \end{array} \right|$$

→ Even Power der Matrix
9062395123
Series / Pattern
Sind für
Progression
A A² A³ ...

a_{11} a_{12} a_{13}
 a_{31} a_{33}

Ratio of 3



The possible set of eigen values of a 4×4 skew-symmetric orthogonal real matrix is (1 mark)

(A) $\{\pm i\}$ (B) $\{\pm i, \pm 1\}$ (C) $\{\pm i\}$ (D) $\{0, \pm i\}$

For E.V. \rightarrow 0 degree wt \rightarrow Unit modulus

$$\sqrt{1-a} \quad a > 0$$

$$\frac{a < 0}{a = 0}$$

$\sin^2 = \cos^2$

Σ v. \rightarrow x degree x \rightarrow Unit modulus $\frac{a=0}{n=y}$
 Σ v. of S.S \rightarrow 0 / purely imaginary $2i+3$ $2i, -2i$ $\frac{a=0}{n=y}$

36. Let P be a 2×2 complex matrix such that $\text{trace}(P)=1$ and $\det(P)=-6$. Then, trace of $(P^4 - P^3)$ is (1 mark)

~~78~~ $\lambda^2 - \lambda - 6 = 0$
 $\lambda^2 - (\text{tr})\lambda + (\det) = 0$
 $\lambda = 3, -2$
 $\text{tr}(P^4 - P^3)$
 $P^4 - P^3 =$ $\begin{pmatrix} 57 & 24 \\ 3 & 4 \end{pmatrix}$
 $57 + 24 = 78$

38. Let M be the real vector space of 2×3 matrices with real entries. Let $T: M \rightarrow M$ be defined by

$$T\left(\begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \end{bmatrix}\right) = \begin{bmatrix} -x_6 & x_4 & x_1 \\ x_3 & x_5 & x_2 \end{bmatrix}. \text{ The determinant of } T \text{ is } \underline{\hspace{2cm}} \quad (2 \text{ marks})$$

41. Let X be the space of all 4×3 matrices with entries in the field of three elements. Then the number of matrices of rank three in M is **(2 marks)**

(A) $(3^4 - 3)(3^4 - 3^2)(3^4 - 3^1)$
(C) $(3^4 - 1)(3^4 - 3)(3^4 - 3^2)$

(B) $(3^4 - 1)(3^4 - 2)(3^4 - 3)$
(D) $3^4(3^4 - 1)(3^4 - 2)$

42. Let V be a vector space of dimension $m \geq 2$. Let $T: V \rightarrow V$ be linear transformation such that $T^{n+1} = 0$ and $T^n \neq 0$ for some $n \geq 1$. Then which of the following is necessary TRUE? (2 marks)
- (A) $\text{Rank}(T^n) \leq \text{Nullity}(T^n)$ (B) $\text{trace}(T) \neq 0$
(C) T is diagonalizable (D) $n=m$

4/2 570 100 20 note | 151 | 60
 373 $M_3(\mathbb{R})$

$32 = 16 \times 2 \times 1 = 19$
 $32 = 4 \times 8 \times 1 = 13 \checkmark$
 $32 =$

43. Let $A \in M_3(\mathbb{R})$ be such that $\det(A-I) = 0$, where I denotes the 3×3 identity matrix. If $\text{trace}(A) = 13$ and $\det(A) = 32$, then the sum of squares of the eigen values of A is (1 mark)

All are 0
 except
middle
diagonal

$$\begin{pmatrix} 8 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$8^2 + 4^2 + 1^2 = 81$

Antony

(83) $\xrightarrow{123456789132234781}$ (3)

(78) $\xrightarrow{19345678914451}$ (2)

N/A
 M/A

$9^1 \rightarrow 9^1$
 $9^2 \rightarrow 81$
 $9^3 \rightarrow 729$

(9) $\xrightarrow{2023}$ (9)
 (2023) $\xrightarrow{2023}$ (9)

(9) $\xrightarrow{2023}$ (9) $\xrightarrow{2023}$ (9) $\xrightarrow{2023}$ (9)
 cycle of (4)

(85434) $\xrightarrow{9291}$

(7) $\xrightarrow{9291}$ (7) $\xrightarrow{91}$ (3)

$4^3 \rightarrow$ (4)

$3 \checkmark$
 $9 \checkmark$
 $27 \checkmark$
 $81 \checkmark$
 $243 \checkmark$
 $729 \checkmark$
 $2187 \checkmark$

49. Let $T_1, T_2: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be linear transformations such that $\text{rank}(T_1) = 3$ and $\text{nullity}(T_2) = 3$. Let $T_3: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear transformation such that $T_3 \circ T_1 = T_2$. Then $\text{rank}(T_3)$ is (2 marks)

52. Let M be a 3×3 matrix and suppose that 1, 2 and 3 are the eigenvalues of M . If $M^{-1} = \frac{M^2}{\alpha} - M + \frac{11}{\alpha}I$, for some scalar $\alpha \neq 0$, then α is equal to 6 (1 mark)

Soln

det $\Rightarrow 6$

$M^{-1} \rightarrow \begin{pmatrix} 1 & 1/2 & 1/3 \\ & & \end{pmatrix}$

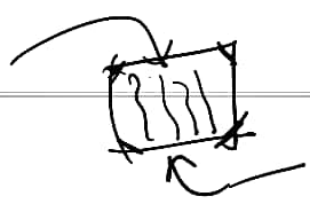
$M^2 \rightarrow \begin{pmatrix} 1 & 4 & 9 \\ & & \end{pmatrix}$

$I \rightarrow \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$ * * *

I of $M \rightarrow M^{-1} = I \quad M^2 = I, I = I$

$I = \frac{M^2}{\alpha} - M + \frac{11}{\alpha}I$

$2 = \frac{12}{\alpha} \Rightarrow \alpha = 6$



ScalM \rightarrow P.S. (Eigenvalues and Eigenvectors)

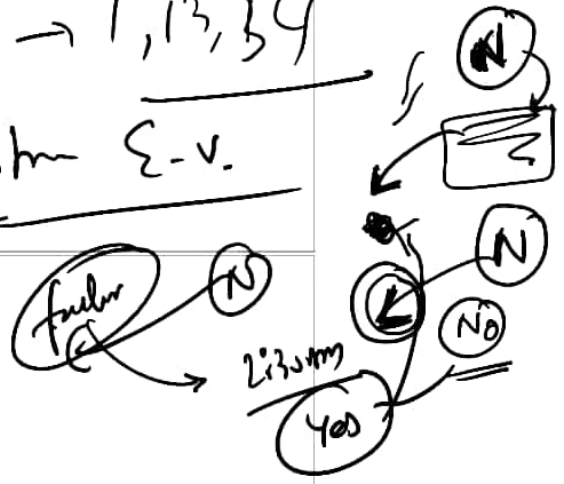
of let \rightarrow Comm \rightarrow V
Old Term \rightarrow

~~Handwritten scribbles at the top of the page.~~

53. Let M be a 3×3 singular matrix and suppose that 2 and 3 are eigenvalues of M . Then the number of linearly independent eigenvectors of $M^3 + 2M + I$, is equal to (1 mark)

Handwritten work for question 53:

- Initial eigenvalues: $0, 2, 3$ (with 0 circled)
- Transformation: $M^3 \rightarrow 0, 2^3, 3^3 \rightarrow 0, 8, 27$
- Transformation: $2M \rightarrow 0, 4, 6$
- Transformation: $M^3 + 2M + I \rightarrow 1, 13, 34$
- Final eigenvalues: $1, 13, 34$ (all circled)
- Conclusion: "Distin ϵ -v." (Distinct eigenvalues)



54. Let M be a 3×3 matrix such that $M \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ -3 \end{pmatrix}$ and suppose that $M \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$ for some

Handwritten work for question 54:

- Eigenvalues: $\alpha_1, \alpha_2, \alpha_3$
- Matrix representation: $M = \begin{pmatrix} -2 & 0 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & -3 \end{pmatrix}$
- Transformation: $M \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ -3 \end{pmatrix}$
- Additional notes: $-2 \rightarrow 6, 1 \rightarrow -3, 0 \rightarrow -3$

$$M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -6 \\ -1 \\ 0 \end{bmatrix}$$

$$M^3 = \begin{bmatrix} -27 & 0 & 0 \\ 0 & -27 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -27 \\ 27 \\ 0 \end{bmatrix}$$

$+ 27 \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \leftarrow R$

57. Let M be an invertible Hermitian matrix and let $x, y \in \mathbb{R}$ be such that $x^2 < 4y$. Then. (2 marks)
- (A) both $M^2 + xM + yI$ and $M^2 - xM + yI$ are singular
 - (B) $M^2 + xM + yI$ is singular but $M^2 - xM + yI$ is non-singular
 - (C) $M^2 + xM + yI$ is non-singular but $M^2 - xM + yI$ is singular
 - (D) both $M^2 + xM + yI$ and $M^2 - xM + yI$ are non-singular

New Semes

Max Test
9862395723

71. Let $A = (a_{ij})$ be a 10×10 matrix such that $a_{ij} = 1$ for $i \neq j$ and $a_{ii} = \alpha + 1$, where $\alpha > 0$. Let λ and μ be the largest and the smallest eigenvalues of A , respectively. If $\lambda + \mu = 24$, then α equals _____
(2 marks)

72. Let $A = \begin{bmatrix} a & 2f & 0 \\ 2f & b & 3f \\ 0 & 3f & c \end{bmatrix}$, where a, b, c, f are real numbers and $f \neq 0$. The geometric multiplicity of the largest eigenvalue of A equals _____.

(1 mark)

73. Consider the subspaces $W_1 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 = x_2 + 2x_3\}$
 $W_2 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 = 3x_2 + 2x_3\}$
of \mathbb{R}^3 . Then the dimension of $W_1 + W_2$ equals _____.

(1 mark)

78. Consider the matrix $A = I_9 - 2u^T u$ with $u = \frac{1}{3} [1, 1, 1, 1, 1, 1, 1, 1, 1]$, where I_9 is the 9×9 identity matrix and u^T is the transpose of u . If λ and μ are two distinct eigenvalues of A , then $|\lambda - \mu| =$ _____.
(2 marks)

79. If the characteristic polynomial and minimal polynomial of a square matrix A are $(\lambda-1)(\lambda+1)^2(\lambda-2)^3$ and $(\lambda-1)(\lambda+1)(\lambda-2)$, respectively, then the rank of the matrix $A + I$ is _____, where I is the identity matrix of appropriate order. (1 mark)

82. Let V be the vector space of all 3×3 matrices with complex entries over the real field. If $W_1 = \{A \in V : A = \overline{A}^T\}$ and $W_2 = \{A \in V : \text{trace of } A = 0\}$, then the dimension of $W_1 + W_2$ is equal to _____ . (2 marks)
- (\overline{A}^T denotes the conjugate transpose of A)

86. Suppose V is a finite dimensional non-zero vector space over \mathbb{C} and $T : V \rightarrow V$ is a linear transformation such that $\text{Range}(T) = \text{Null space}(T)$. Then which of the following statements is FALSE? (2 marks)
- (A) The dimension of V is even
(B) 0 is the only eigenvalue of T
(C) Both 0 and 1 are eigenvalues of T
(D) $T^2 = 0$