LINEAR ALGEBRA SESSION FOR 09.11.23

Linear Algebra Session

WITC: find a pattern.

30. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then A^{50} is (1 mark)

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 50 & 0 & 0 \\ 50 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 48 & 0 & 0 \\ 48 & 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 24 & 1 & 0 \\ 24 & 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

2, 4, 8

$A^8 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$

$A^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$

Even Power

9062395123

A, A^2, A^3

Rule of 3

The possible set of eigen values of a 4×4 skew – symmetric orthogonal real matrix is (1 mark)

(A) {±i} (B) {±i,±1} (C) {±1} (D) {0,±i}

SSM EV → 0 or purely imaginary → Unit modulus

a > 0

a < 0

a = 0

Eg E.V. → Orthogonal mat → Unit modulus

E.v. of S.S → 0 / purely imaginary

$2i + 3$

$2i, -2i$

$\bar{a} = 0$

Sin x = Sin

x = y

Sinh x = Sinh y

x = y

36. *Let P be a 2×2 complex matrix such that trace(P)=1 and det(P)=-6. Then, trace of $(P^4 - P^3)$ is* **(1 mark)**

78

$tr(P^4 - P^3)$

$$\lambda^2 - \lambda - 6 = 0$$

$$\lambda^2 - (Tr)\lambda + (det) = 0$$

$$\lambda = 3, -2$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$\Sigma = Tr$

EV → 54, 24

$P^4 - P^3 =$

$54 + 24 = 78$

38. *Let M be the real vector space of* 2×3 *matrices with real entries. Let* $T: M \to M$ *be defined by* $T\left(\begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \end{bmatrix}\right) = \begin{bmatrix} -x_6 & x_4 & x_1 \\ x_3 & x_5 & x_2 \end{bmatrix}$. *The determinant of T is* ______ ***(2 marks)***

41. *Let X be the space of all* 4×3 *matrices with entries in the field of three elements. Then the number of matrices of rank three in M is* **(2 marks)**

(A) $(3^4-3)(3^4-3^2)(3^4-3^3)$

(B) $(3^4-1)(3^4-2)(3^4-3)$

(C) $(3^4-1)(3^4-3)(3^4-3^2)$

(D) $3^4(3^4-1)(3^4-2)$

42. Let V be a vector space of dimension $m \geq 2$. Let $T: V \rightarrow V$ be linear transformation such that $T^{n+1} = 0$ and $T^{n} \neq 0$ for some $n \geq 1$. Then which of the following is necessary TRUE? (2 marks)

(A) $Rank(T^{n}) \leq Nullity(T^{n})$

(B) $trace(T) \neq 0$

(C) T is diagonalizable

(D) $n = m$

4 | 2 100 20 Gate | ISI (60) 570

a/.

$M_3(R)$

3×3

$32 = 16 \times 2 \times 1 = 19$
$32 = 4 \times 8 \times 1 = 13$ ✓
$32 =$

43. Let $A \in M_3(\mathbb{R})$ be such that $\det(A - I) = 0$, where I denotes the 3×3 identity matrix. If trace $(A) = 13$ and $\det(A) = 32$, then the sum of squares of the eigen values of A is ______ 81

(1 mark)

$$\begin{bmatrix} 8 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{bmatrix} \quad \begin{matrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 2 \end{matrix}$$

$8^2 + 4^2 + 1^2 = 81$

$(83)^{123456789132134781} \to 3$

$(78)^{1934567891441} \to 2$

2023

$(9)^{11} \to 9$

3, 9, 27, 81, 243, 729, 2387

$9^1 = 9$, $9^2 = 81$, $9^3 = 729$

$(2023)^{11} \to (9)^{2023}$ cycle of 4 $\to 9$

$2023^3 \to (7)^9 \to 7^1 \to 7$

$(85434)^{9291}$

9291 → 91 → 3

$4^3 \to 4$

[redacted]

49. Let $T_1, T_2 : \mathbb{R}^5 \to \mathbb{R}^3$ be linear transformations such that rank$(T_1) = 3$ and nullity$(T_2) = 3$. Let $T_3 : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T_3 \circ T_1 = T_2$. Then rank (T_3) is ________

(2 marks)

[redacted] (2 marks)

Det ⇒ 6

[redacted] 52. Let M be a 3×3 matrix and suppose that 1,2 and 3 are the eigenvalues of M. If $M^{-1} = \frac{M^2}{\alpha} - M + \frac{11}{\alpha} I_3$ for some scalar $\alpha \neq 0$, then α is equal to ___6___. (1 mark)

$M^{-1} \rightarrow 1, 1/2, 1/3$

$M^2 \rightarrow 1, 4, 9$

$I \rightarrow 1, 1, 1$

1 of M → $M^{-1} = 1$, $M^2 = 1$, $I = 1$

$1 = \frac{1}{\alpha} - 1 + \frac{11}{\alpha}(1)$ ⇒ $2 = \frac{12}{\alpha}$

$\alpha = 6$

Old Term

E.S

Scom → P.S.

53. Let M be a 3×3 singular matrix and suppose that 2 and 3 are eigenvalues of M. Then the number of linearly independent eigenvectors of $M^3 + 2M + I$, is equal to________. **(1 mark)**

54. Let M be a 3×3 matrix such that $M\begin{pmatrix}-2\\1\\0\end{pmatrix}=\begin{pmatrix}6\\-3\\0\end{pmatrix}$ and suppose that $M^3\begin{pmatrix}1\\-\frac{1}{2}\\0\end{pmatrix}=\begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix}$ for some

$$M = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 0 \end{bmatrix}$$

$$M^3 = \begin{bmatrix} -27 & 0 & 0 \\ 0 & -27 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1/2 \\ 2 \end{bmatrix}$$

$$= -27, \quad +27/2, \quad 0$$

57. *Let M be an invertible Hermitian matrix and let $x, y \in \mathbb{R}$ be such that $x^2 < 4y$. Then.* **(2 marks)**

(A) both $M^2 + xM + yI$ and $M^2 - xM + yI$ are singular

(B) $M^2 + xM + yI$ is singular but $M^2 - xM + yI$ is non-singular

(C) $M^2 + xM + yI$ is non-singular but $M^2 - xM + yI$ is singular

(D) both $M^2 + xM + yI$ and $M^2 - xM + yI$ are non-singular

71. Let $A = (a_{ij})$ be a 10×10 matrix such that $a_{ij} = 1$ for $i \neq j$ and $a_{ii} = \alpha + 1$, where $\alpha > 0$. Let λ and μ be the largest and the smallest eigenvalues of A, respectively. If $\lambda + \mu = 24$, then α equals____

(2 marks)

72. Let $A=\begin{bmatrix} a & 2f & 0 \\ 2f & b & 3f \\ 0 & 3f & c \end{bmatrix}$, where a, b, c, f are real numbers and $f \neq 0$. The geometric multiplicity of the largest eigenvalue of A equals _________ . (1 marks)

73. Consider the subspaces $W_1 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 = x_2 + 2x_3\}$

$$W_2 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 = 3x_2 + 2x_3\}$$

of $\mathbb{R}^3$. Then the dimension of $W_1 + W_2$ equals ________. *(1 marks)*

78. Consider the matrix $A = I_9 - 2u^T u$ with $u = \frac{1}{3}[1, 1, 1, 1, 1, 1, 1, 1, 1]$, where I_9 is the 9×9 identity matrix and u^T is the transpose of u. If λ and μ are two distinct eigenvalues of A, then $|\lambda - \mu| =$ ______.

(2 marks)

79. *If the characteristic polynomial and minimal polynomial of a square matrix A are* $(\lambda-1)(\lambda+1)^4(\lambda-2)^5$ *and* $(\lambda-1)(\lambda+1)(\lambda-2)$, *respectively, then the rank of the matrix* $A + I$ *is _____, where* I *is the identity matrix of appropriate order.* **(1 mark)**

82. *Let V be the vector space of all 3×3 matrices with complex entries over the real field. If $W_1 = \{A \in V : A = \overline{A}^T\}$ and $W_2 = \{A \in V : \text{trace of } A = 0\}$, then the dimension of $W_1 + W_2$ is equal to______.* **(2 marks)**

($\overline{A}^T$ denotes the conjugate transpose of A)

86. Suppose V is a finite dimensional non-zero vector space over $\mathbb{C}$ and $T : V \rightarrow V$ is a linear transformation such that Range(T) = Null space (T). Then which of the following statements is FALSE? **(2 marks)**

(A) The dimension of V is even

(B) 0 is the only eigenvalue of T

(C) Both 0 and 1 are eigenvalues of T

(D) $T^2 = 0$