

②

Let X_1, X_2, \dots, X_n be random variable whose marginal distributions are $N(0,1)$. Suppose $E(X_i X_j) = 0$ for $i, j, i \neq j$. Let $Y = X_1 + X_2 + \dots + X_n$ and $V = X_1^2 + X_2^2 + \dots + X_n^2$. which of the following statement follow from the give conditions?

- X (a) Y has normal distribution with mean zero and variance n
- X (b) V has Chi-square distributon with n degrees of freedom
- X (c) $E(X_i^2 X_j^2) = 0$ for all $i, j, i \neq j$
- (d) $P(X_i > t) \leq \frac{n}{t^2}$ for all $t > 0$ (True)

X_1, X_2, \dots, X_n
 If X_i 's are independent

- ① $\sum X_i \sim N(0, n)$
 But indep not proved
- ② $X_i \sim N(0,1) \Rightarrow Z_i \sim \chi^2_{1.d.f}$
- ③ $E(X_i X_j) = 0 \quad \forall i \neq j$
 $E^3(X_i X_j) = 0^3$
 $E(X_i^2 X_j^2) \neq 0$

df \geq eq - var = ①
 $3 - 4 = -1$
 $df \geq 0$

$Y = X_1 + X_2 + \dots + X_n$

CBV method

$P[|Y - E(Y)| > t] \leq \frac{Var(Y)}{t^2}$

$Var(Y) = \sum Var(X_i) + 2 \{Cov(X_i X_j)\}$

$Cov(X_i X_j) = E(X_i X_j) - E(X_i)E(X_j)$
 $= 0$

$Var(Y) = \sum Var(X_i) = 1 + 1 + \dots + n \text{ times}$
 $= n$

Let X_1, X_2, \dots be i.i.d random variables having a $\chi^2 -$ distribution with 5 degree of freedom. Let $a \in \mathbb{R}$ be constant. Then the limiting distribution of $a \left(\frac{X_1 + \dots + X_n - 5n}{\sqrt{n}} \right)$

$E(X_i) = 5$
 $Var(X_i) = 2 \text{ df} = 10$
 Sum of indep χ^2
 $\dots \sim$ value

... of the random variables having a distribution with 5 degree of freedom. Let $a \in \mathbb{R}$ be constant. Then the limiting distribution of $a \left(\frac{X_1 + \dots + X_n - 5n}{\sqrt{n}} \right)$ is

- (a) Gamma distribution for an appropriate value of a
- (b) χ^2 -Distribution for an appropriate value of a
- (c) Standard normal distribution for an appropriate value of a
- (d) A degenerate distribution for an appropriate value of a

Sum of independent χ^2 variables
 is also χ^2 variable
 $df = \sum$ of their df

$S_n = X_1 + X_2 + \dots + X_n \sim \chi^2_{5n}$

$E(S_n) = 5n$

$Var(S_n) = 10n$

Using Central Limit Theorem

$\frac{S_n - E(S_n)}{\sqrt{Var(S_n)}} \rightarrow N(0, 1)$

$\frac{X_1 + X_2 + \dots + X_n - 5n}{\sqrt{10n}} \sim N(0, 1)$

Then $a \left(\frac{X_1 + \dots + X_n}{\sqrt{n}} \right) = \frac{(X_1 + X_2 + \dots + X_n) - 5n}{\sqrt{10n}} \rightarrow N(0, 1)$

if we let $a = \frac{1}{\sqrt{10}} \in \mathbb{R}$

Suppose X_1, X_2, \dots, X_n are i.i.d. random variables with characteristic function $\phi(t; \theta) = E[e^{itX_i}]$ where $\theta \in \mathbb{R}^k$ is the parameter of the distribution. Let $Z = X_1 + X_2 + \dots + X_n$. Then for which of the following distributions of X_1 would the characteristic function of Z be of the form $\phi(t; \theta)$ for some $\theta \in \mathbb{R}^k$?

- (a) Negative Binomial
- (b) Geometric
- (c) Hypergeometric
- (d) Discrete Uniform

10 Binomial $\rightarrow 3/4 \dots 10$

Given and success calculated

-ve B INOMIAL
 In order to get 5 success how
 ... then you can

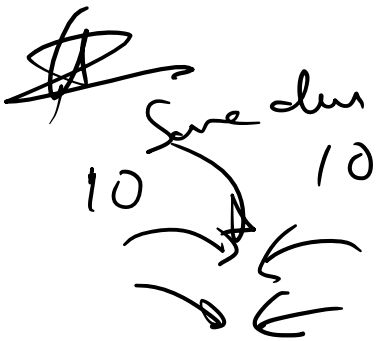
In order to you many times you can $\phi(t)$

Let, $Z = X_1 + X_2 + \dots + X_n$

$$E[e^{itZ}] = E[e^{it(X_1 + X_2 + \dots + X_n)}]$$

$$= E[e^{itX_1}] E[e^{itX_2}] \dots E[e^{itX_n}] = \phi(t, \alpha)$$

as $X_1, X_2, X_3 \dots$ are i.i.d random variables



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Suppose X_1, X_2, \dots, X_n are i.i.d. random variables with characteristic function $\phi(t; \theta) = E[e^{itX_1}]$ where $\theta \in \mathbb{R}^k$ is the parameter of the distribution. Let $Z = X_1 + X_2 + \dots + X_n$. Then for which of the following distributions of X_1 would the characteristic function of Z be of the form $\phi(t; \alpha)$ for some $\alpha \in \mathbb{R}^k$?

- (a) Negative Binomial
- (b) Geometric
- (c) Hypergeometric
- (d) Discrete Uniform

A random variable T has a symmetric distribution if T and $-T$ have the same distribution. Let X and Y be independent random variables. Then which of the following statements are correct?

- (a) If X and Y have the same distribution then $X - Y$ has a symmetric distribution
- (b) If $X \sim N(3, 1)$ and $Y \sim N(2, 2)$, then $2X - 3Y$ has a symmetric distribution

$$\phi_{X-Y}(t) = E[e^{it(X-Y)}]$$

$$= E[e^{itX} e^{-itY}]$$

$$= E[e^{itX}] E[e^{-itY}]$$

$$= \phi_X(it) \phi_{-Y}(t)$$

- (b) If $X \sim N(3,1)$ and $Y \sim N(2,2)$, then $2X - 3Y$ has a symmetric distribution
- (c) If X and Y have the same symmetric distribution, then $X + Y$ has a symmetric distribution
- (d) If X has a symmetric distribution, then XY has a symmetric distribution

$$= \phi_x(t) \cdot \phi_{-y}(t)$$

$X \sim Y, Y \sim X \rightarrow$ same dist

$$X \sim -X \quad \phi_X(t) = \phi_{-X}(t)$$

$$Y \sim -Y \quad \phi_Y(t) = \phi_{-Y}(t)$$

$$\phi_{X+Y}(t) = \phi_X(t) \cdot \phi_Y(t)$$

$$= \phi_{-X}(t) \phi_{-Y}(t)$$

$$= \phi_{-(X+Y)}(t)$$

(d)

Symmetric dist

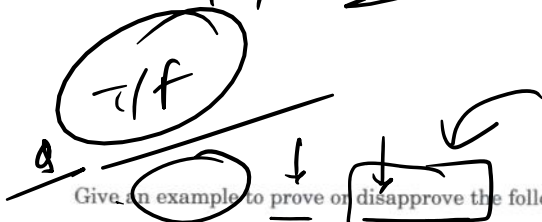
$$f(x) = f(-x)$$

$$f(x+y) = f(x)f(y)$$

$$\frac{f(-x)f(-y)}{f(-x-y)} = \frac{f(-x) \cdot f(-y)}{f(-x-y)}$$

$$= f(-x-y)$$

x, y are symmetric



$$\frac{4+n}{n} \rightarrow \left(\frac{4}{n} + 1\right)$$

$$n+n = 2n$$

Give an example to prove or disapprove the following :

Give an example to prove or disapprove the following:

$$P(\limsup A_n) = 0 \Rightarrow \sum_{k=1}^{\infty} P(A_k) < \infty$$

for any sequence $\{A_n, n \geq 1\}$ of events defined on a probability space (Ω, \mathcal{A}, P) .

$n+n = \sup$

$$\begin{matrix} 2.33 \\ 2 \end{matrix} \downarrow$$

requires AX

$P(\limsup A_n) = 0 \rightarrow$ finite sums

INF (1)
 $U_n = n$
 1 2 ... ∞

note 1 Sup can be achieved at the
 have pt of the prob space or not we never know

$\Omega = \{1, 2, \dots\}$

$A_n = \{n\}$ $n \in \Omega$
 $P(\{n\}) = 1/n$ $n \in \Omega$

$P(\limsup A_n) = P(\Omega) = 1$