04 April 2024 18:50

2

Let $X_1, X_2, \ldots X_n$ be random variable whose marginal distributions are $N(0,1)$. Suppose $E(X_i X_j) =$ for $i, j, i \neq j$. Let $Y = X_1 + X - 2 + \cdots + X_n$ and $V = X_1^2 + X_2^2 + .. + X_n^2$. which of the following statement follow from the give conditions?

(a) Y has normal distribution with mean zero and variance n

(b) V has Chi-square distributon with n degrees of freedom

(c) $E(X_i^3 X_j^3) = 0$ for all $i, j, i \neq j$

(d) $P(|Y| > t) \leq \frac{n}{t^2}$ for all $t > 0$ (True)

$X_1, X_2 \ldots X_n$

If X_i's are independent

1 $\sum X_i \sim N(0, n)$

But independence not formed

2 $X_i \sim N(0,1) \Rightarrow Z_i^2 \sim \chi^2_{1 \cdot df}$

3 $E(X_i X_j) = 0 \quad \forall i \neq j$

$E^3(X_i X_j) = 0^3$

$E(X_i^3 Y_i^3) \neq 0$

$df \Rightarrow eq - unk = 1$

$3 - 4 = -1$

$\boxed{df \geq 0}$

4 $Y = X_1 + X_2 + \cdots + X_n$

CBV inequality

$P[|Y - E(Y)| > t] \leq \frac{Var(Y)}{t^2}$

$V(Y) = \sum V(X_i) + 2\sum Cov(X_i X_j)$

$Cov(X_i X_j) = E(X_i X_j) - E(X_i)E(X_j)$

$= 0$

$Var(Y) = \sum V(X_i) = 1 + 1 + \cdots n$ times

$= n$

$E(X_1) = 5$

$V(X_1) = 2 \cdot df = 10$

values

5 * Sum of indep χ^2

χ^2 variable

(NOV 2020)

Let $X_1, X_2, \ldots$ be i.i.d random variables having a χ^2- distribution with 5 degree of freedom. Let $a \in \mathbb{R}$ be constant. Then the limiting distribution of $a\left(\frac{X_1 + .. + X_n - 5n}{\sqrt{\cdot}}\right)$

... be i.i.d random variables having ... χ^2 distribution with 5 degree of freedom. Let $a \in R$ be constant. Then the limiting distribution of $a\left(\frac{X_1+..+X_n-5n}{\sqrt{n}}\right)$ is

(a) Gamma distribution for an appropriate value of a
(b) χ^2 –Distribution for an appropriate value of a
(c) Standard normal distribution for an appropriate value of a $= \frac{1}{\sqrt{10}}$
(d) A degenerate distribution for an appropriate value of a

Sum of indep χ^2 is also χ^2 with df = $\sum$ sum of their df

$S_n = X_1 + X_2 + \dots + X_n \sim \chi^2_{5n}$

$E(S_n) = 5n$

$Var(S_n) = 10n$

Using Central Limit Theorem..

$$\frac{S_n - E(S_n)}{\sqrt{Var(S_n)}} \to N(0,1)$$

$$\frac{X_1 + X_2 \dots + X_n - 5n}{\sqrt{10n}} \sim N(0,1)$$

Then $a\left(\frac{X_1 + \dots + X_n}{\sqrt{n}}\right) = \frac{(X_1 + X_2 + \dots + X_n) - 5n}{\sqrt{10n}} \to N(0,1)$

when $a = \frac{1}{\sqrt{10}} \in R$

Suppose $X_1, X_2, \dots, X_n$ are i.i.d. random variables with characteristic function $\phi(t;\theta) = E[e^{itX_i}]$ where $\theta \in R^k$ is the parameter of the distribution. Let $Z = X_1 + X_2 + \dots + X_n$. Then for which of the following distributions of X_1 would the characteristic function of Z be of the form $\phi(t;\alpha)$ for some $\alpha \in R^k$?

(a) Negative Binomial (b) Geometric
(c) Hypergeometric (d) Discrete Uniform

10 Binomial $\to$ 3/4 ... 10

Given trial success calculated..

–ve BINOMIAL

In order to get 5 success how many times you can ...

In order to ... you ...
many times you can (try)

Let, $Z = X_1 + X_2 + \dots + X_n$

$$E[e^{itZ}] = E[e^{it(X_1+X_2+\dots+X_n)}]$$

$$E[e^{itX_1}]\,E[e^{itX_2}]\cdots E[e^{itX_n}] = \phi(t,\alpha)$$

$$\to \phi(t,\alpha)$$

as $X_1, X_2, X_3 \dots X_n$ are iid random variables

Same dus

10 10

ISI 2020

Suppose $X_1, X_2, \dots, X_n$ are i.i.d. random variables with characteristic function $\phi(t;\theta) = E[e^{itX_i}]$ where $\underline{\theta} \epsilon R^k$ is the parameter of the distribution. Let $Z = X_1 + X_2 + \dots + X_n$. Then for which of the following distributions of X_1 would the characteristic function of Z be of the form $\phi(t;\underline{\alpha})$ for some $\underline{\alpha} \epsilon R^k$?

(a) Negative Binomial (b) Geometric
(c) Hypergeometric (d) Discrete Uniform

A random variable T has a symmetric distribution if T and −T have the same distribution. Let X and Y be independent random variables. Then which of the following statements are correct?

(a) If X and Y have the same distribution then X – Y has a symmetric distribution
(b) If X ~ N(3,1) and Y ~ N(2,2), then 2X − 3Y has a symmetric distribution

$$\phi_{X-Y}(t) = E[e^{X-Y}]$$
$$= E(e^{itX} e^{-itY})$$
$$= E(e^{itX})\,E(e^{-itY})$$
$$= \phi_X(t)\,\phi_{-Y}(t)$$

(b) If X ~ N(3,1) and Y ~ N(2,2), then 2X − 3Y has a symmetric distribution

(c) If X and Y have the same symmetric distribution, then X + Y has a symmetric distribution

(d) If X has a symmetric distribution, then XY has a symmetric distribution

$= \phi_x(t) \cdot \phi_{-y}t$

X−Y, Y−X → same dist

$X, -X \quad \phi_X(t) = \phi_{-(x)}t$

$Y, -Y \longrightarrow \phi_Y(t) = \phi_{-Y}(t)$

$\phi_{X+Y}(t) = \phi_X(t) \cdot \phi_Y(t)$

$= \phi_{-X}(t)\phi_{-Y}(t)$

$= \phi_{-(X+Y)}t$

symmetric dist

d)

$f(x) = f(-x)$

$f(xy) = f(x)f(y)$

$\boxed{f(xy) =} f(-x)\cdot f(y)$

$= f(-xy)$

X, Y, A are symmetric

T/F

$\frac{4+n}{n} \to \left(\frac{4}{n}+1\right)$

n+n^2 ... 5 3

Give an example to prove or disapprove the following :

Give an example to prove or disapprove the following :

$$P(\limsup A_n) = 0 \Rightarrow \sum_{k=1}^{\infty} P(A_k) < \infty,$$

for any sequence $\{A_n, n \geq 1\}$ of events defined on a probability space $(\Omega, \mathcal{A}, P)$.

15

$n \in N$ = sups

3

2.33

2

INF 1

$U_n = n$

1, 2 - - - - - ∞

inf

$P(\limsup A_n) = 0 \to$ finite sums

Note:1 Sup can be achieved at the

base pt of the prob space or at the nearest ..

$\Omega = \{1, 2, \dots\}$

$A_n = \{n\}$ $\quad n \in \mathbb{Q}$

$P(n) = 1/n$ $\quad n \in \mathbb{Q}$ ..

$P(\limsup A_n) = P(\Omega) = 1$