Result: The monopolist always operates on the elastic portion of the demand curve ($\epsilon > 1$)

ϵ = abs. price elasticity of demand $= -\dfrac{\%\Delta q}{\%\Delta p} = -\dfrac{d[\ln q]}{d[\ln p]}$.

Multiplant Monopolist:

Consider a monopolist selling his output in 1 market and has 2 production facilities/plants to produce that output. Let q_1 and q_2 denote the output level in Plant 1 & Plant 2.

$\therefore$ Cost fn for Plant 1: $C_1 = C_1(q_1)$, $C_1' > 0$.

Cost fn for Plant 2: $C_2 = C_2(q_2)$, $C_2' > 0$.

Total output of the monopolist: $q = q_1 + q_2$.

Inverse demand curve for monopolist is $P(q)$, $P' < 0$.

Monopolist will decide on opt levels of q_1 & q_2 based on π-max.

$$\pi = P.q - C_1(q_1) - C_2(q_2) \quad ; \quad q = q_1 + q_2$$

$$\pi = \underbrace{P(q_1+q_2)(q_1+q_2)} - C_1(q_1) - C_2(q_2).$$

Total Revenue $= R(q_1+q_2) = R(q)$

$$\pi = R(q) - C_1(q_1) - C_2(q_2)$$

FOC:

$$\frac{\partial \pi}{\partial q_1} = 0 \Rightarrow \frac{\partial R}{\partial q}\cdot\underbrace{\left(\frac{\partial q}{\partial q_1}\right)}_{=1} - \frac{\partial C_1}{\partial q_1} = 0 \Rightarrow MR = MC_1 \text{ --- (i)}$$

$q = q_1 + q_2$

$\dfrac{\partial q}{\partial q_1} = 1$

$$\frac{\partial \pi}{\partial q_2} = 0 \Rightarrow \frac{\partial R}{\partial q}\left(\frac{\partial q}{\partial q_2}\right) - \frac{\partial C_2}{\partial q_2} = 0 \Rightarrow MR = MC_2 \text{ ---- (ii)}$$

(i) & (ii) will give: $q_1^*, q_2^* \Rightarrow q^* = q_1^* + q_2^*$

(i) & (ii) will give: q_1^*, q_2^* $\Rightarrow$ $q^* = q_1^* + q_2^*$

Replace q^* in the demand eqn to get $p^* = p(q^*)$

(optimal price charged by the monopolist)

Combining (i) & (ii): $\boxed{MR = MC_1 = MC_2}$ $\Rightarrow$ Optimization condition.

Q. Consider a monopolist producing in 2 plants given by: $MC_1(q_1) = 20 + 2q_1$ and $MC_2(q_2) = 10 + 5q_2$. The mkt demand is given by $P = 20 - 3q$. Find the π-max production levels in both the plants.

$MC_1 = 20 + 2q_1$

$C_1 = \int 20 + 2q_1 \, dq_1$

$= 20q_1 + q_1^2$

$$q = q_1 + q_2 \Rightarrow P = 20 - 3(q_1 + q_2).$$

$$\pi = [20 - 3(q_1 + q_2)](q_1 + q_2) - (20 + \cancel{2}q_1)q_1 - (10 + \tfrac{5}{2}q_2)q_2.$$

$$\frac{\partial \pi}{\partial q_1} = 0 \Rightarrow (q_1 + q_2)(-3) + [20 - 3(q_1 + q_2)] \cdot 1 - (20 + 2q_1) = 0.$$

$$\Rightarrow 20 - 6(q_1 + q_2) = 20 + 2q_1 \quad \text{.....} (i)$$

$$\frac{\partial \pi}{\partial q_2} = 0 \Rightarrow (q_1 + q_2)(-3) + [20 - 3(q_1 + q_2)] \cdot 1 - (10 + 5q_2) = 0.$$

$$\Rightarrow 20 - 6(q_1 + q_2) = 10 + 5q_2 \quad \text{....} (ii)$$

From (i) & (ii): $20 + 2q_1 = 10 + 5q_2$,

$$10 + 2q_1 = 5q_2 \Rightarrow q_2 = 2 + \frac{2}{5}q_1.$$

Put (i): $20 - 6(q_1 + 2 + \frac{2}{5}q_1) = 20 + 2q_1$.

$$\cancel{20} - 6q_1 - 12 - \frac{12}{5}q_1 = \cancel{20} + 2q_1$$

$$-12 = 6q_1 + \frac{12}{5}q_1 + 2q_1.$$

$$q_1^* = -\frac{15}{13} < 0$$

$MR = 20 - 6q.$

$MC_1 = 20 + 2q_1$

$q_1^* = -\frac{15}{13} < 0$

$\downarrow$

q_1^* is infeasible.

Hence monopolist will produce in Plant 2.

$\therefore$ Use (ii): $20 - 6(q_1 + q_2) = 10 + 5q_2$.

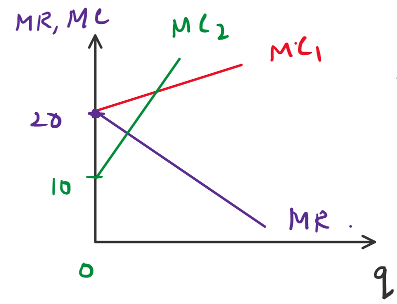
Put $q_1 = 0 \Rightarrow 20 - 6q_2 = 10 + 5q_2$

$q_2^* = \frac{10}{11}$. ✓

$MR = 20 - 6q$.

$MC_1 = 20 + 2q_1$

$MC_2 = 10 + 5q_2$



Note: For a multiplant monopolist opt condition is $MR = MC_1 = MC_2$ only when the monopolist produces in both the plants. If for eg: the monopolist produces only in Plant 2 then the opt condition is $MR = MC_2$, where put $q_1 = 0$ & solve for q_2^*.

Multimarket Monopolist: [Third-Degree Price Discrimination].

Consider a monopolist selling his product in 2 mkts and producing in 1 plant with cost fn: $C = c(q)$, $c' > 0$, $q = q_1 + q_2$.

where q_1 = output sold in Mkt 1, q_2 = output sold in Mkt 2

Let demand fn for mkt 1: $P_1 = P_1(q_1)$, $P_1' < 0$.

& demand fn for mkt 2: $P_2 = P_2(q_2)$, $P_2' < 0$.

Monopolist decides opt choices: q_1^* & $q_2^* \Rightarrow P_1^*, P_2^*$

$\pi = R_1(q_1) + R_2(q_2) - C(q)$, $q = q_1 + q_2$.

$\frac{\partial \pi}{\partial q_1} = 0 \Rightarrow \frac{\partial R_1}{\partial q_1} - \frac{\partial C}{\partial q} \cdot \left(\frac{\partial q}{\partial q_1}\right) = 0 \Rightarrow MR_1 = MC$ --- (i)

$\frac{\partial \pi}{\partial q_2} = 0 \Rightarrow \frac{\partial R_2}{\partial q_2} - \frac{\partial C}{\partial q}\left(\frac{\partial q}{\partial q_2}\right) = 0 \Rightarrow MR_2 = MC$ --- (ii)

$q = q_1 + q_2$

$\frac{\partial q}{\partial q_1} = 1 = \frac{\partial q}{\partial q_2}$.

$\therefore$ Using (i), (ii) solve for q_1^* & q_2^*

$\therefore$ Using (i), (ii) solve for q_1^* & q_2^*

& replacing them in the demand curves: $\boxed{\begin{array}{l} P_1^* = P_1(q_1^*) \\ P_2^* = P_2(q_2^*) \end{array}}$

Opt prices charged by the monopolist in the 2 mkts.

Combining (i) & (ii): $\boxed{MR_1 = MR_2 = MC}$ $\Rightarrow$ Opt condition.

Q. How does the monopolist decide in which mkt he can charge higher price?

HW. Formulate exp to show elasticity of demand will determine in which mkt monopolist can charge higher price.