Consumer Behavior [Demand side Analysis]:

Consider a 2 good framework: Consumer will have a choice to choose level of consumption Good 1, Good 2.

Good 1 ↳ (x_1), Good 2 ↳ (x_2)

Utility fn: $u = u(x_1, x_2)$; $\boxed{\frac{\partial u}{\partial x_1} > 0, \ \frac{\partial u}{\partial x_2} > 0}$.

[Digression: suppose there was 1 Good ⇒ Good x.

Utility fn: $u = u(x)$ ⇒ $\frac{du}{dx}$: change in u due to change in x

If x increases by 1 units, by how much does u increase – Marginal utility of Good x

Consider 2 goods Good 1, Good 2

Utility fn: $u = u(x_1, x_2)$, $\frac{\partial u}{\partial x_1} > 0$, $\frac{\partial u}{\partial x_2} > 0$,

$\frac{\partial u}{\partial x_1}$ = Diff u w.r.t x_1 keeping x_2 constant $= MU_1$

$\frac{\partial u}{\partial x_2}$ = Diff u w.r.t x_2 keeping x_1 constant $= MU_2$.

Note: $\frac{\partial u}{\partial x_1} < 0$ ..... irrational behavior (cannot be considered)

Eg: $u(x_1, x_2) = x_1^2 + x_2^2 + x_1 x_2$. Find MU_1, MU_2:

$MU_1 = \frac{\partial u}{\partial x_1} = 2x_1 + x_2$, $MU_2 = \frac{\partial u}{\partial x_2} = 2x_2 + x_1$.

(*) All rules for differentiation are applicable for partial derivatives as well].

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[It captures the preference pattern of the individual]

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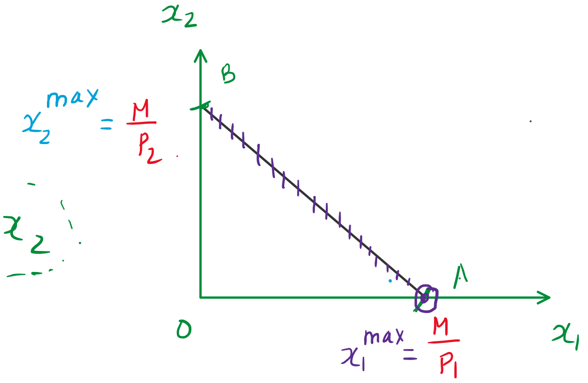
Budget constraint:

Money Income $= M$.
Mkt Price of Good 1 $= P_1$
Mkt Price of Good 2 $= P_2$
} Parameters for consumers

Variables: x_1, x_2



Budget constraint: $\boxed{M = P_1 x_1 + P_2 x_2}$

Diff: $0 = P_1\, dx_1 + P_2\, dx_2$.

$P_2\, dx_2 = -P_1\, dx_1$.

$\frac{dx_2}{dx_1} = -\frac{P_1}{P_2}$ ---- Slope of B·L.

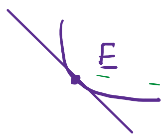
For pt A, put $x_2 = 0$.
$M = P_1 x_1 \Rightarrow x_1 = \frac{M}{P_1}$.
For pt B, put $x_1 = 0$.
$M = P_2 x_2 \Rightarrow x_2 = \frac{M}{P_2}$.

Absolute slope of BL: $\left|\frac{dx_2}{dx_1}\right| = \frac{P_1}{P_2}$.

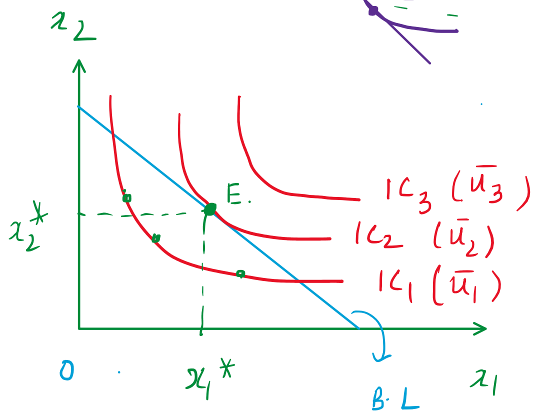
$\left(\frac{dx_2}{dx_1}\right) = -\frac{P_1}{P_2} < 0$ ..... If the consumer wants to increase his consumption of Good 1 by 1 unit then he has to sacrifice $\left(\frac{P_1}{P_2}\right)$ units of Good 2
[Relative price of Good 1 (in terms of Good 2)]

Obj of the consumer:- [Max utility subject to Budget constraint]

$$\max_{x_1, x_2} \; u = u(x_1, x_2) \quad s.t \quad M = P_1 x_1 + P_2 x_2$$



At the optimal, pt E, $x_1^*, x_2^* > 0$.
E (interior solution)



Case I: Interior solution

[Highest possible IC meets the BL] .

At pt E, Slope of IC = Slope of B·L.

Slope of IC:

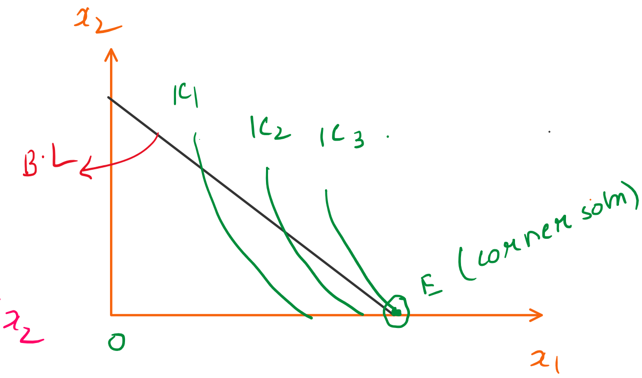
$$u = u(x_1, x_2)$$

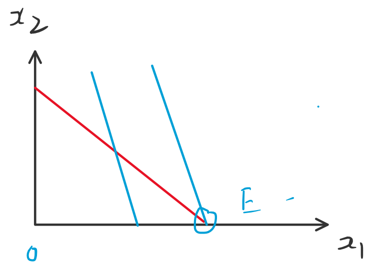
Diff: $$du = \frac{\partial u}{\partial x_1} \cdot dx_1 + \frac{\partial u}{\partial x_2} \cdot dx_2 .$$

For IC, $du = 0$. $\Rightarrow$ $$0 = \left(\frac{\partial u}{\partial x_1}\right) dx_1 + \left(\frac{\partial u}{\partial x_2}\right) dx_2$$

$$\Rightarrow \quad 0 = MU_1 \cdot dx_1 + MU_2 \cdot dx_2 .$$

$$\Rightarrow \quad \left.\frac{dx_2}{dx_1}\right|_{IC} = -\frac{MU_1}{MU_2}$$





$\therefore$ At pt E, slope of IC = slope of BL

$$\Rightarrow \cancel{-} \frac{MU_1}{MU_2} = \cancel{-} \frac{P_1}{P_2} \quad \Rightarrow \quad \boxed{\frac{MU_1}{MU_2} = \frac{P_1}{P_2}}$$

Note: The optimization condition: $\frac{MU_1}{MU_2} = \frac{P_1}{P_2}$ is only valid for interior solutions.