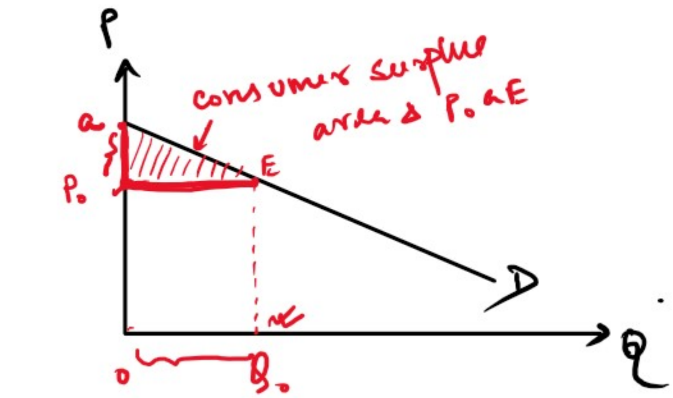
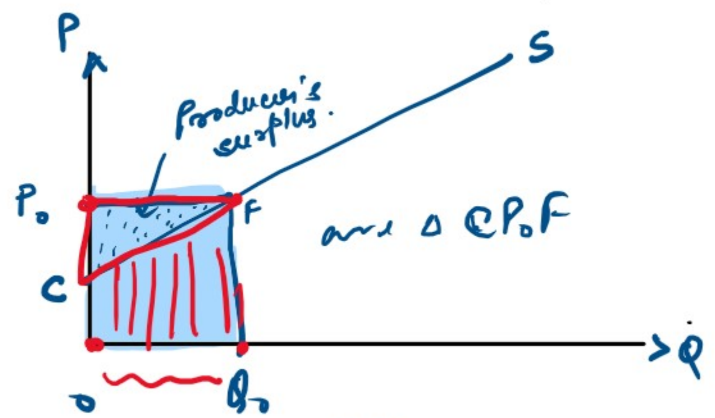
29 February 2024 07:02 PM

Consumer and Producer's Surplus.





D: $\boxed{P = a - bQ}$

max willingness to pay (a); slope of demand (b)

S: $c + dQ$

min acceptance price (c); slope of supply curve (d)

$CS = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times OQ_0 \times (Oa - OP_0)$

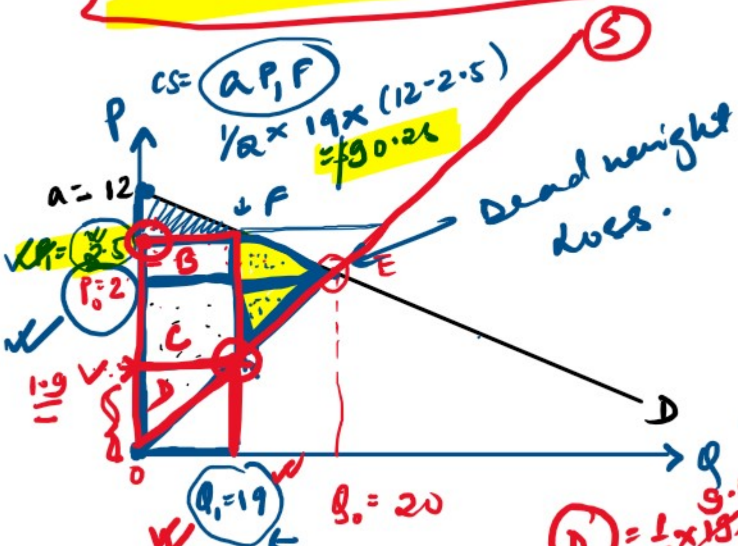
OR

$$\boxed{CS = \int_0^{Q_0} D(Q)\, dQ - OP_0 \times OQ_0}$$

$PS = \frac{1}{2} \times CP_0 \times OQ_0$

$\to \frac{1}{2} \times (OP_0 - OC) \times (OQ_0)$

$$\boxed{PS = (OP_0 \times OQ_0) - \int_0^{Q_0} S(Q)\, dQ}$$



$$\left\{ \begin{array}{l} D: \ P = 12 - 0.5Q_D \\ S: \ P = 0.1Q_S \end{array} \right\} \text{Linear}$$

(a) $P^* = \$2$

$Q^* = 20$

$CS = \frac{1}{2} \times (Oa - OP_0) \times OQ_0$

$= \frac{1}{\cancel{2}} \times (12 - 2) \times \overset{10}{\cancel{20}}$

$= 10 \times 10 = 100$

B $= (2.5 - 2) \times 19 = 9.5$

C $= (2 - 1.9) \times 19 = 1.9$

D $= \frac{1}{\cancel{2}} \times \overset{9.5}{\cancel{19}} \times 1.9 = 18.05$

$PS = \frac{1}{2} \times OQ_0 \times OP_0$

OR

$CS = \int_{20}^{Q_0} D(Q)\, dQ - P_0 \times Q_0$

$PS = \frac{1}{2} \times OQ_0 \times OP_0$

$= \frac{1}{\cancel{2}} \times 20 \times \cancel{2} = \$20$ ✓

OR

$$PS = (OQ_0 \times PQ_0) - \int_0^{20} S(Q)\,dQ$$

$$= (20 \times 2) - \int_0^{20} (0.1Q)\,dQ$$

$$= 40 - 0.1\left[\frac{Q^2}{2}\right]_0^{20}$$

$$= 40 - \frac{0.1}{2}[400]$$

$$= 40 - 20$$

$= \$20$ (ans) ✓

OR

$$CS = \int_0^{20} D(Q)\,dQ - P_0 \times Q_0$$

$$= \int_0^{20} (12 - 0.5Q)\,dQ - (2 \times 20)$$

$$= \left[12Q - 0.5\frac{Q^2}{2}\right]_0^{20} - 40$$

$$= 12(20) - \frac{0.5}{2}(20^2) - 40$$

$$= 240 - \frac{0.5 \times \cancel{400}^{200}}{\cancel{2}} - 40$$

$$= 240 - 100 - 40$$

$$= 140 - 40$$

$$\boxed{CS = 100}$$ ✓

SW = CS + PS = area OQE

After price support at $P_1 = \$2.5$

$CS_1 = \$90.25$

and producer's surplus, PS_1 = area (B + C + D)

$= 9.5 + 1.9 + 18.05$

$= \$29.45$

$\Delta CS = 100 - 90.25 = 9.75$ ✓ (decrease in CS due to min support price)

$= 29.45 - 20 = 9.45$ ✓ (increase in)

to min ... price)

$\Delta PS = 29.45 - 20 = 9.45$ (increase in PS)

$\therefore$ Loss in SW $= 9.45 - 9.75$
$= (\$\,0.30)$
dead weight loss.

Topic : Elasticity of Demand :

own price elasticity

$$e_p^x = \frac{\%\text{ change in Quantity demand}}{\%\text{ change in Price}}$$

$$e_p^x = \frac{\frac{\Delta Q}{Q}\times 100}{\frac{\Delta P}{P}\times 100}$$

$$= \frac{\Delta Q}{\Delta P}\times\frac{P}{Q} \text{ or } \boxed{\frac{dQ}{dP}\times\frac{P}{Q}}$$

in case of Normal good change Q in inversely related to change in p

$$\therefore \boxed{|e_p^x| = \left|\frac{dQ}{dP}\cdot\frac{P}{Q}\right|}$$

income elasticity

$$e_M^x = \frac{\%\text{ change in Quantity demand}}{\%\text{ change in income}}$$

$$= \frac{\frac{\Delta Q}{Q}\times 100}{\frac{\Delta M}{M}\times 100}$$

$$= \frac{\Delta Q}{\Delta M}\times\frac{M}{Q} \text{ or } \boxed{\frac{dQ}{dM}\times\frac{M}{Q}}$$

cross price elasticity

$$e_c^{x,y} = \frac{\%\text{ change in } Q_x}{\%\text{ change in } P_y}$$

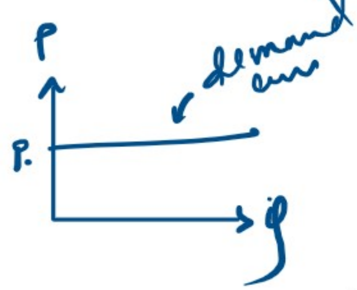
$$= \frac{\frac{\Delta Q^x}{Q^x}\times 100}{\frac{\Delta P_y}{P_y}\times 100}$$

$$= \frac{\Delta Q_x}{\Delta P_y}\times\frac{P_y}{Q_x} \text{ or } \boxed{\frac{dQ_x}{dP_y}\times\frac{P_y}{Q_x}}$$

$\frac{dP}{} \frac{}{Q}$

Own Price elasticity:

① perfectly elastic demand ($|e_p^x| \to \infty$)



$|e_p^x| \to \infty$ [Q, P ≥ 0]

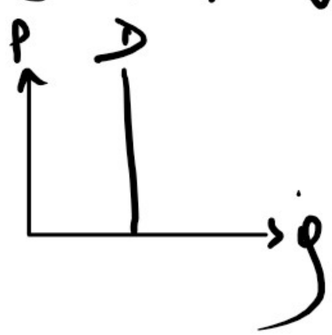
$\frac{\Delta Q}{\Delta P} \to \infty$

$\Delta P \to 0$

ie, for any negligible change in P, q demand is highly responsive

∴ demand curve is horizontal to the Quantity axis.

② perfectly inelastic demand $|e_p^x| \to 0$



$\frac{\Delta Q}{\Delta P} \to 0$

$\Delta Q \to 0$

ie for any change in price q Quantity demand remain unchanged.

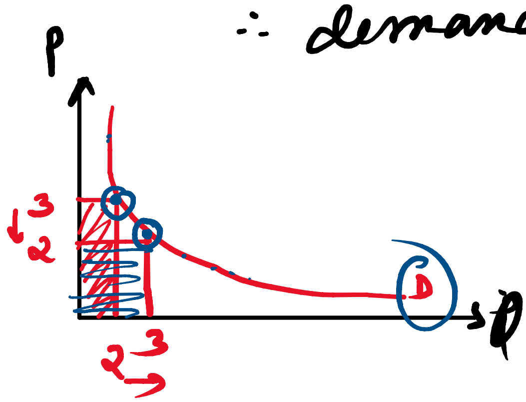
q demand curve is vertical

③ $|e_p^d| = 1$ (ie unit elastic).

% change in Q = % change in P

∴ demand curve is rectangular

∴ demand curve is $\boxed{\text{rectangular hyperbola}}$.



TR is constant.

(4) Relatively elastic $|e_p^x| > 1$ and relatively inelastic $|e_p^x| < 1$

Elastic: $|e_p^x| > 1$

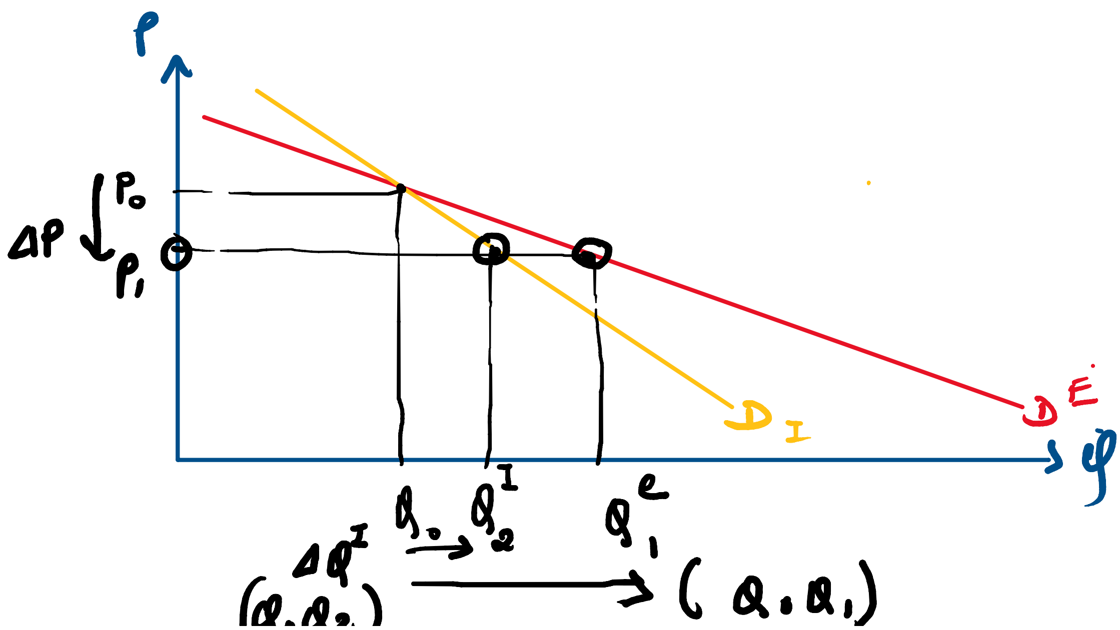
% change in Q > % change in P

Demand curve is flatter. (more respon)

Inelastic: $|e_p^x| < 1$

% change in Q < % change in P (less respon)

Demand curve is steeper.



$$\Delta Q^{1} \quad (Q_0 Q_2) \longrightarrow (Q_0 Q_1)$$

① $|e_p| > 1$ elastic demand — $P\uparrow \Rightarrow TR = P \times Q$ (decrease); $P\downarrow \Rightarrow TR = P \times Q$ (increase)

② $|e_p| < 1$ inelastic demand — $P\uparrow \Rightarrow TR = P \times Q$ (increase); $P\downarrow \Rightarrow TR = P \times Q$ (decrease)

③ $|e_p| = 1$ unit elastic — $P\uparrow$ or $P\downarrow$ (TR remain same)

Relation between TR, AR, MR and $|e_p^x|$

$$TR = P \times Q$$

$$AR = \frac{TR}{Q} = \frac{P \times \cancel{Q}}{\cancel{Q}} = P$$

$$MR = \frac{dTR}{dQ} = P + Q\frac{dP}{dQ}$$

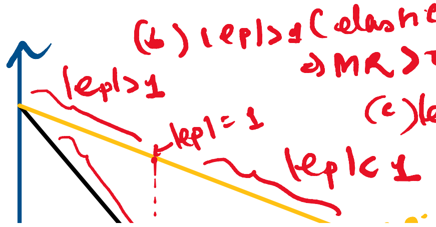
$$MR = P\left[1 + \frac{Q}{P} \cdot \frac{dP}{dQ}\right]$$

AR, MR

(a) $|e_p| = 1$ (unit elastic) $\Rightarrow MR = 0$

(b) $|e_p| > 1$ (elastic) $\Rightarrow MR > 0$

(c) $|e_p| < 1$ (unit elastic) $\Rightarrow MR < 0$



$$MR = P\left[1 + \frac{1}{\frac{P}{Q}\cdot\frac{dQ}{dP}}\right]$$

$$MR = AR\left[1 - \frac{1}{-\frac{P}{Q}\cdot\frac{dQ}{dP}}\right]$$

$$\boxed{MR = AR\left[1 - \frac{1}{|e_P|}\right]}$$

Q: For the demand function $q = 30 - 4p - p^2$

(i) find the elasticity of demand when $p = 3$.

(ii) Find MR when $p = 3$.

Soln:

$$q = 30 - 4p - p^2$$

$$e_p^d = \frac{dq}{dp} \cdot \frac{p}{q}$$

$$\frac{dq}{dp} = -4 - 2p$$

$; \; p = 3$

$\therefore q = 30 - 4\times3 - 3^2 = 30 - 12 - 9 = 9$

$$\therefore e_p^d = (-4-2p) \times \frac{3}{9}$$

$$= (-4 - 2\times3) \times \frac{1}{3}$$

$$= \underline{-10}$$

3

$= -3.3$

$|e_p^d| = 3.3 > 1$ (demand is elastic).

(ii) $MR = ?$

$AR = P = 3$

$(e_p^d) = 3.3$.

$\therefore MR = AR\left[1 - \frac{1}{|e|}\right]$

$= 3\left[1 - \frac{1}{3.3}\right]$

$= 3\left[\frac{3.3-1}{3.3}\right]$

$= 3\left[\frac{2.3}{3.3}\right]$

$= 2.1$ (ans)

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