To test: $H_0: \beta = 0$ vs $H_1: \beta \neq 0$.

Note: After performing the test if we reject $H_0 \Rightarrow X$ variable is meaningful as an explanatory variable, i.e. "X is significant".

Recap: PRF: $Y_i = \alpha + \beta X_i + u_i$, $u_i \sim$ Gauss Markov Assumptions.

(*) In addition assume $u_i \overset{iid}{\sim} N(0, \sigma^2)$.

Result: Any linear combination of normal variates also follow a normal distribution with corresponding mean & variance.

Note: as $\hat{\beta}$ is a linear combination of u_i's:

$$\hat{\beta} \sim N\left[E(\hat{\beta}), Var(\hat{\beta})\right].$$

$$\Rightarrow \hat{\beta} \sim N\left[\beta, \frac{\sigma^2}{\sum x_i^2}\right].$$

Eg: $X_1 \sim N(\mu_1, \sigma_1^2)$

$X_2 \sim N(\mu_2, \sigma_2^2)$

$X_3 = a_1 X_1 + a_2 X_2$

$\therefore X_3 \sim N\left[E(X_3), Var(X_3)\right]$

$$\therefore \frac{\hat{\beta} - \beta}{\sigma / \sqrt{\sum x_i}} \sim N(0,1)$$

[This cannot be used as a test-statistic, as σ is unknown].

Result: $$\frac{\sum e_i^2}{\sigma^2} \sim \chi^2_{(n-2)}$$

$\left[\sum \left(\frac{e_i}{\sigma}\right)^2 = \frac{e_1^2}{\sigma^2} + \frac{e_2^2}{\sigma^2} + \cdots + \frac{e_n^2}{\sigma^2} \Rightarrow n\text{-terms} \right.$

No. of restrictions $= 2$ $(\sum e_i = 0, \sum e_i X_i = 0)$

$\left. \therefore df = (n-2) \right]$

$$(\&) \Rightarrow \frac{(\hat{\beta} - \beta)^2}{\sigma^2 / \sum x_i^2} \sim \chi^2_{(1)}$$

independent.

$$F = \frac{\chi^2_{(1)}/1}{\chi^2_{(n-2)}/(n-2)} \sim F_{1,(n-2)}$$

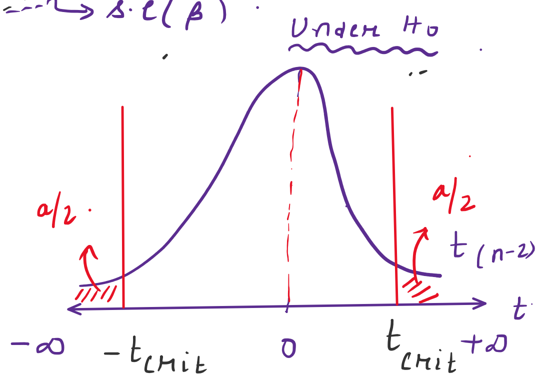
$$= \frac{\dfrac{(\hat\beta-\beta)^2}{\cancel{\sigma^2}/\sum x_i^2}}{\dfrac{\sum e_i^2}{\cancel{\sigma^2}}/(n-2)} = \frac{(\hat\beta-\beta)^2\sum x_i^2}{\underbrace{\left(\dfrac{\sum e_i^2}{n-2}\right)}_{=\hat\sigma^2}} = \frac{(\hat\beta-\beta)^2}{\hat\sigma^2/\sum x_i^2} \sim F_{1,(n-2)}.$$

$$\therefore \sqrt{\frac{(\hat\beta-\beta)^2}{\hat\sigma^2/\sum x_i^2}} \sim t_{(n-2)} \Rightarrow \frac{\hat\beta-\beta}{\underbrace{\hat\sigma/\sqrt{\sum x_i^2}}_{s.e(\hat\beta)}} \sim t_{(n-2)}$$

Test statistic.

Test statistic: $t = \dfrac{\hat\beta-\beta}{s.e(\hat\beta)} \sim t_{(n-2)}$.

Under H_0: $t = \dfrac{\hat\beta}{s.e(\hat\beta)} \sim t_{(n-2)}$



Testing Rule: We will reject H_0 at $\alpha\%$ L.O.S.
if $t < -t_{crit}$ or $t > t_{crit} \Rightarrow |t| > t_{crit}$

ANOVA Analysis: [ANOVA = Analysis of Variance].

Obj: PRF: $Y_i = \alpha + \beta X_i + u_i$ [$\Rightarrow$ X causes Y].

Inference — Forecasting / Prediction.

$\sum(Y_i-\bar Y)^2$ = Total variation in Y.

$$\sum(Y_i-\bar Y)^2 = \sum\left\{\underbrace{(Y_i-\hat Y_i)}_{=e_i} + (\hat Y_i - \bar Y)\right\}^2$$

$$= \sum(Y_i-\hat Y_i)^2 + \sum(\hat Y_i-\bar Y)^2 + 2\sum(Y_i-\hat Y_i)(\hat Y_i-\bar Y)$$

$$= \sum e_i^2 + \sum(\hat Y_i - \bar{\hat Y})^2 + 2\underbrace{\sum e_i(\hat Y_i-\bar Y)}_{0} \quad [\bar Y = \bar{\hat Y}].$$

$$\boxed{\sum(Y_i-\bar Y)^2 = \sum(\hat Y_i-\bar{\hat Y})^2 + \sum e_i^2}$$

$$\boxed{\sum (Y_i-\bar{Y})^2 = \sum(\hat{Y}_i-\bar{\hat{Y}})^2+\sum e_i^2} \quad \text{--- } (*)$$

Now, $\sum e_i(\hat{Y}_i-\bar{Y}) = \sum e_i \hat{Y}_i - \bar{Y}\sum e_i = 0$.

$$= \sum e_i(\hat{\alpha}+\hat{\beta}x_i) = \hat{\alpha}\underbrace{\sum e_i}_{=0} + \hat{\beta}\underbrace{\sum e_i x_i}_{=0}$$

Now, $\sum(Y_i-\bar{Y})^2$ = Total variation in Y = TSS (Total sum of sq).

$\sum(\hat{Y}_i-\bar{\hat{Y}})^2$ = Total variation in $\hat{Y}$ ($\hat{Y}$ is the part of Y that is estimated by X).

= Total variation in Y that can be explained by X = ESS (Explained sum of sq).

$\sum e_i^2$ = Error variation / part of variation that cannot be explained by X = RSS (Residual sum of sq)

$\Rightarrow$ ANOVA: $\boxed{TSS = ESS + RSS}$ --- $(*)$