Recap of Assumptions of the Solow Model:

(i) $Y = F(K, L)$; $F_L > 0$, $F_{LL} < 0$, $F_K > 0$, $F_{KK} < 0$.

(ii) $\lambda \cdot Y = F(\lambda K, \lambda L)$, λ is a scalar ---- (CRS)

(iii) $S = s \cdot Y$, $0 < s < 1$. [s = savings rate for the economy].

(iv) $\frac{\dot{L}}{L} = n$, $n > 0$. [n = growth rate of labour]

(v) $\dot{K} = I - \delta \cdot K$, $\delta > 0$ [δ = depreciation rate]

$\therefore$ Derived prodn fn in intrinsic form/ per capita form:

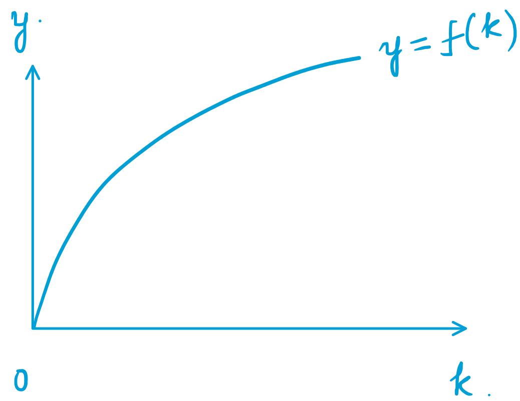
$$y = f(k), \quad y = \left(\frac{Y}{L}\right), \quad k = \left(\frac{K}{L}\right)$$

$\rightarrow$ capital per unit of labour (capital per capita)

$\hookrightarrow$ output per unit of labour (output per capita).

$$y = f(k) \Rightarrow \frac{Y}{L} = f\left(\frac{K}{L}\right).$$

$$\Rightarrow Y = L \cdot f\left(\frac{K}{L}\right).$$



$$F_K = \frac{\partial Y}{\partial K} = L \cdot f'\left(\frac{K}{L}\right)\left(\frac{1}{L}\right) = f'\left(\frac{K}{L}\right) = f'(k)$$

$\therefore$ We know $F_K > 0 \Rightarrow f'(k) > 0$.

$$F_{KK} = \frac{\partial^2 Y}{\partial K^2} = f''\left(\frac{K}{L}\right)\left(\frac{1}{L}\right) = \frac{1}{L} \cdot f''(k).$$

$\therefore$ We know $F_{KK} < 0 \Rightarrow \frac{1}{L} \cdot f''(k) < 0 \Rightarrow f''(k) < 0$.

$\therefore$ Summarizing: $\boxed{y = f(k),\ f' > 0,\ f'' < 0}$

Note: To measure economic growth

$\rightarrow$ Measured by changes in Y.

Note: To measure economic growth

→ Measured by changes in Y overtime

→ Given the framework, changes in Y captured by '$y = \frac{Y}{L}$'

→ Changes in 'y' is captured by changes in k
[$\because y = f(k)$].

∴ To measure economic growth ⇒
evaluate how 'k' changes overtime.

Evaluating the Dynamics of 'k':-

$$k = \frac{K}{L} \Rightarrow \ln k = \ln K - \ln L.$$

Diff w.r.t 't':- $\frac{1}{k}\cdot\left(\frac{dk}{dt}\right) = \frac{1}{K}\left(\frac{dK}{dt}\right) - \frac{1}{L}\cdot\left(\frac{dL}{dt}\right).$

$$\Rightarrow \frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L}$$

$$\Rightarrow \frac{\dot{k}}{k} = \frac{I - \delta K}{K} - n.$$

$$\Rightarrow \frac{\dot{k}}{k} = \frac{I}{K} - (\delta + n).$$

$$\Rightarrow \frac{\dot{k}}{k} = \frac{S}{K} - (\delta + n)$$ [$\because$ Given competitive setup $S = I$]

$$\Rightarrow \frac{\dot{k}}{k} = \frac{s \cdot Y}{K} - (\delta + n)$$

$$\Rightarrow \frac{\dot{k}}{k} = \frac{s\cdot(Y/L)}{(K/L)} - (\delta + n)$$

$$\Rightarrow \frac{\dot{k}}{k} = \frac{s \cdot y}{k} - (\delta + n)$$

$$\Rightarrow \quad \dot{k} = s \cdot y - (\delta + n) \cdot k$$

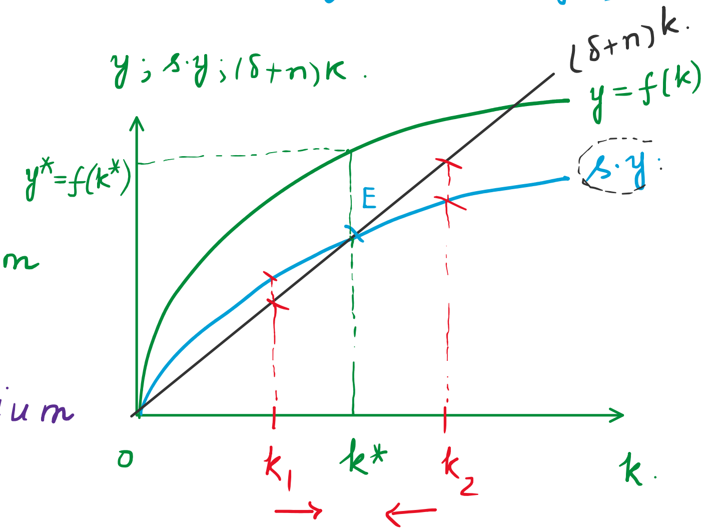
$$\text{or} \quad \boxed{\dot{k} = s \cdot f(k) - (\delta + n) k}$$

↳ Eqn capturing the dynamics of 'k'.

At pt E, $s \cdot y = (\delta + n) k \Rightarrow \dot{k} = 0$.

i.e 'k' stops varying, i.e k achieves its equillibrium value.

Pt 'E' ⇒ Steady state Equillibrium (SSE)



At $k = k_1 \Rightarrow s \cdot y > (\delta + n) k \Rightarrow \dot{k} > 0$

At $k = k_2 \Rightarrow s \cdot y < (\delta + n) k \Rightarrow \dot{k} < 0$

⇒ Pt 'E' is a stable equillibrium.

Implications:

At SSE, $\dot{k} = 0$: Now $y = f(k)$.

$$\Rightarrow \dot{y} = f'(k) \cdot \dot{k}$$

$$\text{As } \dot{k} = 0 \Rightarrow \dot{y} = 0$$

Now, $y = \dfrac{Y}{L} \Rightarrow \ln y = \ln Y - \ln L$.

$$\Rightarrow \frac{\dot{y}}{y} = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}$$

$$\text{At SSE, } \dot{y} = 0 \Rightarrow 0 = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}$$

$$\Rightarrow \left(\frac{\dot{Y}}{Y}\right) = \left(\frac{\dot{L}}{L}\right) = n \Rightarrow \boxed{g_R = n}$$

∴ At SSE, growth rate of agg output (Y) is same as the

$\therefore$ At SSE, growth rate of agg output (Y) is same as the growth rate of labour (L).

Policy implications of the Solow Model:-

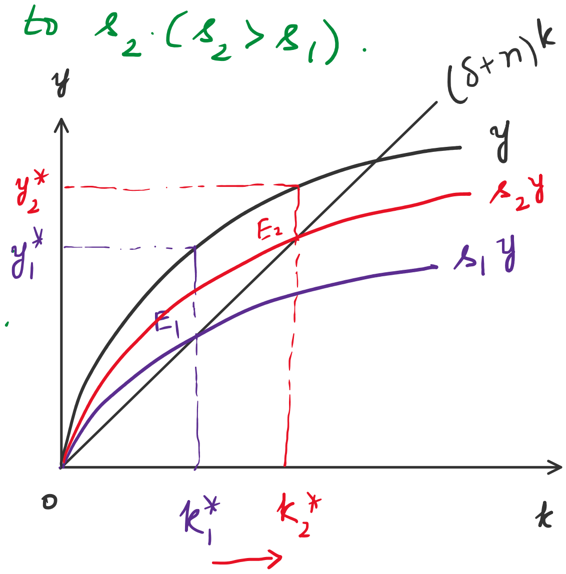
(i) Parametric increase in savings rate (s):

Savings rate increases from s_1 to s_2. $(s_2 > s_1)$.

$$\dot{k} = s \cdot y - (\delta + n)k.$$

Initially $s = s_1 \Rightarrow \dot{k} = s_1 \cdot y - (\delta + n)k$

Finally $s = s_2 \Rightarrow \dot{k} = s_2 \cdot y - (\delta + n)k$.



$\therefore$ Increase in saving rate $\Rightarrow$ Increases $y \Rightarrow$ Increase in Y $\Rightarrow$ Higher economic growth.

HW.

Q. Agg production fn: $Y = K^{\alpha} L^{1-\alpha}$, $0 < \alpha < 1$. [All other assumptions hold]. Find the SSE under the Solow Growth Model.

$k^* =$ $\qquad\qquad$ $y^* =$