Summary of Open sets:

# / Op.	Finite	Infinite
Union	Open set	Open set
Int.	Open set	Not necessarily be open.

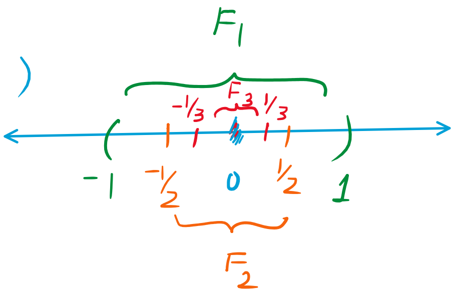
Intersection of infinite number of open sets may not be open.

Eg: $F_n = \left\{ x \in \mathbb{R} \mid -\frac{1}{n} < x < \frac{1}{n} \right\}$, $n = 1, 2, \ldots\ldots$

$F_1 = \{ x \in \mathbb{R} \mid -1 < x < 1 \}$ $(-1, 1)$

$F_2 = \{ x \in \mathbb{R} \mid -\frac{1}{2} < x < \frac{1}{2} \}$ $(-\frac{1}{2}, \frac{1}{2})$

$\ldots\ldots\ldots\ldots$

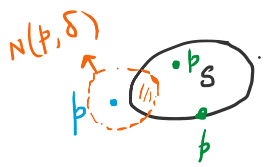


$\bigcap_{n=1}^{\infty} F_n = \{0\} \Rightarrow$ not an open set.

Limit Point:

Let $S \subset \mathbb{R}$. A point $p \in \mathbb{R}$ is said to be a limit point of S if every neighbourhood of p contains a point of S other than p.

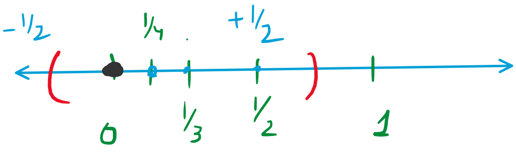
$$\underbrace{[N(p,\delta) - \{p\}]}_{\text{Deleted neighbourhood of } p} \cap S \neq \phi.$$



Note: The pt 'p' may or may not be contained in S.

Eg: $S = \left\{ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\ldots \right\}$. $p = 0 \Rightarrow$ Limit pt ?

$\delta = \frac{1}{2}$



We can choose $\delta > 0$, however small, then also the deleted neighbourhood of $p = 0$ will have atleast one pt of S.

$\therefore$ $p = 0$ is a limit pt of set S.

Theorem: Let $S \subset \mathbb{R}$ and 'p' be a limit pt of set S. Then every neighbourhood of 'p' contains infinitely many pts of S.

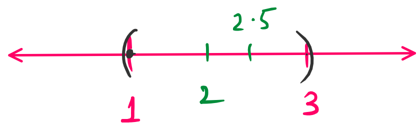
Closed sets.

Let $S \subset \mathbb{R}$. S is said to be a closed set if S contains all its limit points.

Eg: $S = \left\{ 1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots \right\}$. , $p = 0$ is a limit pt.

$p \notin S \Rightarrow S$ is not closed.

$S = (1, 3)$



For any $x \in S$, s.t $1 < x < 3$,

x is a limit pt of set S.

But $x = 1, 3$ are limit pts of set S but $\notin S$

$\therefore$ S is not closed.

$S = [1, 3] \Rightarrow$ closed set.

(*)
Note: Open Interval $\Rightarrow$ Open set. Eg: (a, b)

Closed Interval $\Rightarrow$ Closed set. Eg: $[a, b]$.

Summary of Closed sets:

Summary of Closed sets:

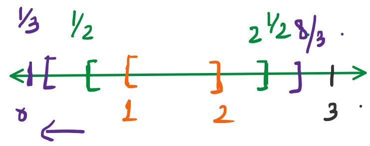
Op. \ #	Finite	Infinite
Union	Closed set.	May not be closed.
Int.	Closed set.	Closed set.

Union of infinite no. of closed sets may not be closed.

Eg: $F_n = \{x \in \mathbb{R} \mid \frac{1}{n} \leq x \leq 3 - \frac{1}{n}\}, \quad n = 1, 2, \ldots$

$F_1 = \{x \in \mathbb{R} \mid 1 \leq x \leq 2\}$.

$F_2 = \{x \in \mathbb{R} \mid \frac{1}{2} \leq x \leq 3 - \frac{1}{2}\}$.



$\bigcup_{n=1}^{\infty} F_n = \{x \in \mathbb{R} \mid 0 < x < 3\} \Rightarrow$ not closed.

HW

Q. $S = (0, 1]$ and $T = \{\frac{1}{n}, n = 1, 2, 3, \ldots\}$

Check if $(S - T)$ is open/closed.