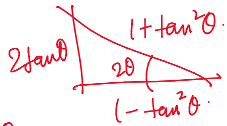
$z = x + iy$ $z = r(\cos\theta + i\sin\theta)$ $z = r.e^{i\theta}$

Exercise 2. $\mathrm{Re}\left\{\dfrac{1+i\tan\left(\frac{\theta}{2}\right)}{1-i\tan\left(\frac{\theta}{2}\right)}\right\}$ *is*

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$



$\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$ $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

(a) $\cos\theta$ *(b)* $\cos\left(\frac{\theta}{2}\right)$ *(c)* $\sin\theta$ *(d)* $\sin\left(\frac{\theta}{2}\right)$

$$\frac{(1+i\tan\theta/2)^2}{1-i^2\tan^2\theta/2} = \frac{1+i^2\tan^2\theta/2 + 2i\tan\theta/2}{1+\tan^2\theta/2}$$

$i^2 = -1$

$$= \underbrace{\left(\frac{1-\tan^2\theta/2}{1+\tan^2\theta/2}\right)}_{Re} + i\cdot\underbrace{\left(\frac{2\tan\theta/2}{1+\tan^2\theta/2}\right)}_{Im}.$$

$Re = \cos\theta$

$Im = \sin\theta$.

Exercise 4. *The solution of the equation* $|z| - z = 1 + 2i$ *is*

(a) $1 - 2i$ *(b)* $2 - \frac{3}{2}i$ *(c)* $\frac{3}{2} + 2i$ *(d)* $\frac{3}{2} - 2i$

When you are dealing with addition/subtraction of complex nos it is generally preferred to use the cartesian form. When you are dealing with multiplication/division of complex nos use polar form.

$z = x + iy$ $|z| = \sqrt{x^2+y^2}$

$\sqrt{x^2+y^2} - (x+iy) = 1+2i$

$(\sqrt{x^2+y^2} - x) - iy = 1 + 2i$

$\boxed{y = -2}$ $\sqrt{x^2+y^2} - x = 1$

$\sqrt{x^2+4} = x+1$

$\cancel{x^2}+4 = \cancel{x^2}+2x+1$

$2x = 3$

$\boxed{x = 3/2}$

$z = \frac{3}{2} - 2i$

Exercise 6. $|z - 2i| + |z + 2i| = 6$ *is a/an*

(a) circle *(b) straight line* *(c) ellipse* *(d) hyperbola*

$|z + ai|$ represents ?

$z = x + iy$ $z + ai = x + i(y+a)$

$r. = |z+ai| = \sqrt{x^2+(y+a)^2}$

$|z + ai|$ represents ?

$z = x + iy$

$r = |z+ai| = \sqrt{x^2+(y+a)^2}$

$x^2 + (y+a)^2 = r^2$. circle: center $(0,-a)$

radius: $|z+ai|$

$x^2+y^2=4$ $\quad (x-1)^2+(y-2)^2=9$

$|x+i(y-2)| + |x+i(y+2)| = 6.$

$\sqrt{x^2+(y-2)^2} + \sqrt{x^2+(y+2)^2} = 6.$

$\sqrt{x^2+(y-2)^2} = 6 - \sqrt{x^2+(y+2)^2}$

$\cancel{x^2}+(y-2)^2 = 36 + \cancel{x^2}+(y+2)^2 - 12\sqrt{x^2+(y+2)^2}$

$12\sqrt{x^2+(y+2)^2} = \cancel{y^2}+4y+\cancel{4}-(\cancel{y^2}-4y+\cancel{4})+36$

$= 8y+36.$

$3\sqrt{x^2+(y+2)^2} = 2y+9$

$9[x^2+(y+2)^2] = 4y^2+36y+81$

$9[x^2+y^2+\cancel{4y}+4] = 4y^2+\cancel{36}y+81$

$9x^2+5y^2 = 45$

$\boxed{\frac{x^2}{5}+\frac{y^2}{9}=1}$

Exercise 7. *The value of* $(\omega^3+\omega^4+\omega^5)\left(\frac{1}{\omega^3}+\frac{1}{\omega^4}+\frac{1}{\omega^5}\right)$, *where ω is cube root of unity, is*

(a) 1 (b) 0 ✓ (c) 2 (d) none of these

$x^3 = 1 \quad x = ?$

$(x^3-1)=0$

$(x-1)(x^2+x+1)=0. \Rightarrow$ Real solution is $x=1$

Complex solution $\boxed{x^2+x+1=0}$ roots ω, ω^2.

$x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$

cube roots of unity are $\omega, \omega^2, 1$

$\boxed{\omega = \frac{-1+\sqrt{3}i}{2}}$

$\boxed{\omega^2+\omega+1=0}$

$\omega^3(\omega^2+\omega+1)=0.$

$\omega^2 = \frac{1+3i^2-2\sqrt{3}i}{4}$

$= \frac{-2-2\sqrt{3}i}{4}$

$= \frac{-1-\sqrt{3}i}{2}$

Exercise 3. *Principal value of* $\log[(1+i)^2]$ *is* _______________.

$\log_e(z) = ?$

$\boxed{\log_e e^{f(x)} = f(x)}$

$z = 1+i = \sqrt{2}e^{i\frac{\pi}{4}}.$

$z = x+iy$

$= r.e^{i\theta}$

$r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$z^2 = 2e^{i\frac{\pi}{2}}.$

$\log(z^2) = \log\left[2e^{i\pi/2}\right] = \log 2 + i\frac{\pi}{2}$

Principal value

$e^{i\theta} = \cos\theta + i\sin\theta.$

$= \cos(\theta+2n\pi) + i\sin(\theta+2n\pi)$

$\log(z) = \log[\ \ldots$

Principal value

Generic solution $= \log 2 + i\left(\frac{\pi}{2} + 2n\pi\right)$

$= \cos(\theta + 2n\pi) + i\sin(\theta + 2n\pi)$

Complex powers of complex Nos.

$z_1^{z_2}$. $z_1, z_2 \in \mathbb{C}$

Exercise 4. Re$\{(1-i)^{1+i}\}$ is______________

Re(z)

$z = (1-i)^{(1+i)}$

$\log z = (1+i)\log(1-i) = \log(1-i) + i\log(1-i)$

$\log z = \log\sqrt{2} + i\left(-\frac{\pi}{4}\right) + i\left[\log\sqrt{2} + i\left(-\frac{\pi}{4}\right)\right]$

$= \frac{1}{2}\log 2 - i\frac{\pi}{4} + i\cdot\frac{1}{2}\log 2 + \frac{\pi}{4}$.

$\log_e z = \left(\frac{1}{2}\log 2 + \frac{\pi}{4}\right) + i\left(\frac{1}{2}\log 2 - \frac{\pi}{4}\right)$

$z = e^{\left[\left(\frac{1}{2}\log 2 + \pi/4\right) + i\left(\frac{1}{2}\log 2 - \frac{\pi}{4}\right)\right]}$

$z = e^{\frac{1}{2}\log 2 + \pi/4} \cdot e^{i\left(\frac{1}{2}\log 2 - \pi/4\right)}$

$z = x + iy = re^{i\theta}$.

$\boxed{\log z = \log r + i\theta}$ principal

$= \log r + i(\theta + 2n\pi)$ generic.

$z = 1 - i$

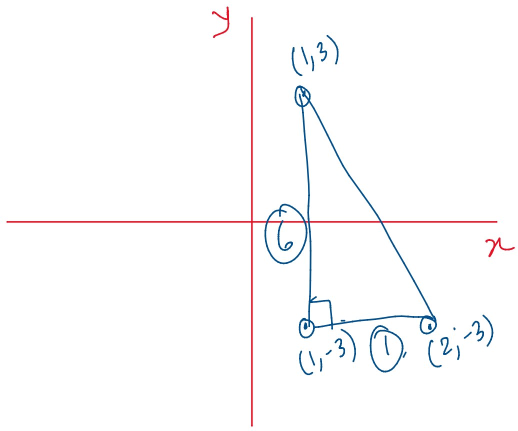
$r = \sqrt{2}$, $\theta = -\frac{\pi}{4}$.

$e^{i\theta} = \underset{\text{Real}}{\cos\theta} + i\underset{\text{Im.}}{\sin\theta}$

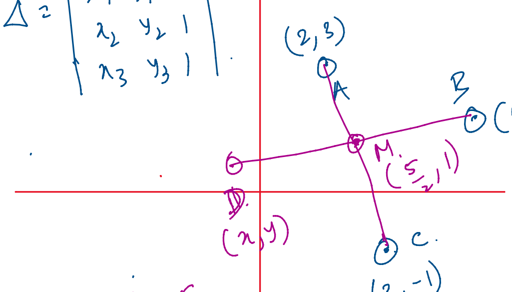
$\text{Re}(z) = e^{\frac{1}{2}\log 2 + \pi/4} \cdot \cos\left(\frac{1}{2}\log 2 - \frac{\pi}{4}\right) = \sqrt{2}\, e^{\pi/4} \cos\left(\frac{1}{2}\log 2 - \frac{\pi}{4}\right)$

Exercise 5. *The points $z_1 = 1 + 3i$, $z_2 = 1 - 3i$ and $z_3 = 2 - 3i$ represents a triangle whose area is*

(a) 3 *(b) 2* *(c) 4* *(d) None of these*



$\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$



$\frac{x+4}{2} = \frac{5}{2}$ $x = 1$

$\frac{y+1}{2} = 1$ $y = 1$

A, B, C → 3 corners of a parallelogram find the 4th vertex

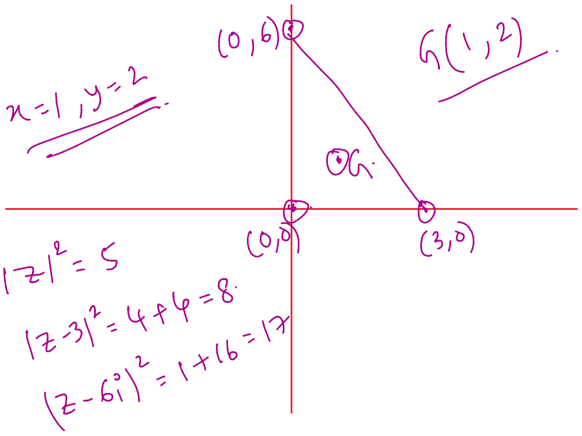
↓

diagonals bisect each other

Example 2. The minimum possible value of $|z|^2+|z-3|^2+|z-6i|^2$, *where* z *is a complex number and* $i=\sqrt{-1}$, *is*

(CSIR UGC NET JUNE-2013)

(a) 15 (b) 45 (c) 30 ✓ (d) 20



$x=1,\ y=2$

$|z|^2=5$

$|z-3|^2=4+4=8$

$|z-6i|^2=1+16=17$

$|z|^2=r_1^2$

$|z-3|^2=r_2^2$

$|z-6i|^2=r_3^2$

$(r_1^2+r_2^2+r_3^2)_{min}$

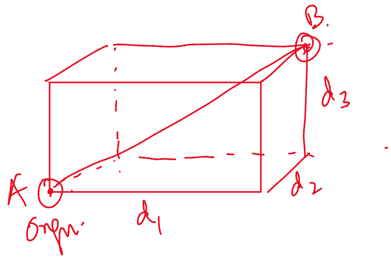
↓

Centroid of the Δ

$|z| \to$ circle with center $(0,0)$

$|z-3| \to$ circle with center $(3,0)$

$|z-6i| \to$ circle with center $(0,6)$



$d_1^2+d_2^2+d_3^2=x$

x_{min} = shortest distance between A and B.

Example 3. Which of the following is the imaginary part of a possible value of $\ell n(\sqrt{i})$?

(a) π (b) $\pi/2$

(c) $\pi/4$ (d) $\pi/8$