CamScanner 09-14-2023 15.39.40

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LIMIT THEOREMS

CamScanner 09-14-2...

$\mu_x = 90$ $\sigma_x = 15$ $n = 25$

$\bar{X} \sim N\left(90, \frac{15}{\sqrt{25}}\right)$ 5

$P(85 < \bar{X} < 92)$

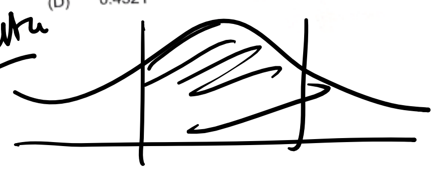
1. An unknown distribution has a mean of 90 and a standard deviation of 15. Samples of size n = 25 are drawn randomly from the population. Find the probability that the **sample mean** is between 85 and 92.

(A) 0.3546 (B) 0.6997

(C) 0.1254 (D) 0.4521

Standard Pattern

20.5



1 applies CLT

$$z = \frac{5.9-6}{\frac{1}{\sqrt{1000}}} = \boxed{-3.16}$$

2. A produce company claims that the mean weight of peaches in a large shipment is 6.0 oz with a standard deviation of 1.0 oz. Assuming this claim is true, what is the probability that a random sample of 1000 of these peaches would have a mean weight of 5.9 oz or less?

(A) 0.0008 (B) 0.654

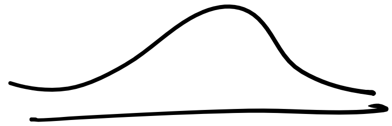
(C) 3.16 (D) 6.0

$$P(Z < -3.16) = 0.5 - 0.4992$$

$$= 0.008$$

t vs z

n ≥ 30 n < 30



3. Let $Y = X_1 + X_2 + \dots + X_{15}$ be the sum of a random sample of size 15 from the distribution whose density function is

$$f(x) = \begin{cases} \frac{3}{2}x^2 & \text{if } -1 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

What is the approximate value of $P(-0.3 \le Y \le 1.5)$ when one use the central limit theorem?

(A) 0.2131 (B) 0.5214

(C) 0.2313 (D) None of the above

$\mu = \int_{-1}^{1} \frac{3}{2}x^3 dx = \frac{3}{2}\left[\frac{x^4}{4}\right]_{-1}^{1} = 0$

Again, $var(X) = E(X^2) - [E(X)]^2$

$= \int_{-1}^{1} \frac{3}{2}x^4 dx = \frac{3}{2}\left[\frac{x^5}{5}\right]_{-1}^{1} = \frac{3}{5} = 0.6$

So, $P(-0.3 \le Y \le 1.5)$

$= P\left(\frac{-0.3}{\sqrt{15(0.6)}} \le \frac{Y}{\sqrt{15(0.6)}} \le \frac{1.5}{\sqrt{15(0.6)}}\right)$

$= P(-0.1 \le Z \le 0.50)$

$= P(Z \le 0.50) + P(Z \le 0.10) - 1$

$= 0.6915 + 0.5398 - 1$

$= 0.2313$

(C) (D) None of the above

4. Let $X_1, X_2, \dots, X_n$ be a random sample of size $n = 25$ from a population that has a mean $\mu = 71.43$ and variance $\sigma^2 = 56.25$. Let $\bar{X}$ be the sample mean. What is the probability that the sample mean is between 68.91 and 71.97?

(A) 0.3654 (B) 0.21465

(C) 0.3654 (D) 0.5941

Same as (3)

5. Light bulbs are installed successively into a socket. If we assume that each light bulb has a mean life of 2 months with a standard deviation on 0.25 months, what is the probability that 40 bulbs last at least 7 years?

(A) 0.0057 (B) 0.57

(C) 0.057 (D) None of the above

$S_{40} = (X_1 + X_2 + \dots + X_{40})$

By CLT $\sum_{i=1}^{40} \frac{X_i - n\mu}{\sqrt{n\sigma^2}} \sim N(0,1)$ as $n \to \infty$

Thus, $\frac{S_{40} - 40.2}{\sqrt{40(0.25)^2}} \sim N(0,1)$

$\frac{S_{40} - 80}{1.581} \sim N(0,1)$

$P(S_{40} \geq 7(12))$

$P\left(\frac{S_{40} - 80}{1.581} \geq \frac{84 - 80}{1.581}\right)$

$= P(Z \geq 2.530) = 0.0057$

$\frac{S_{40} - 80}{1.581}$

$P(S_{40} \geq$ [illegible] $)$

$P\left(\frac{S_{40} - 80}{1.581} \geq \frac{84 - 80}{1.581}\right)$

$= P(Z \geq 2.530) = 0.0057$

6. Light bulbs are installed into a socket. Assume that each has a mean life of 2 months with standard deviation of 0.25 month. How many bulbs n should be bought so that one can be 95% sure that the supply of n bulbs will last 5 years?

(A) 31 (B) 31.15

(C) 31.10 (D) 31.5

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7. American Airlines claims that the average number of people who pay for in-flight moves, when the plane is fully loaded, is 42 with a standard deviation of 8. A sample of 36 fully loaded planes is taken. What is the probability that fewer than 38 people paid for the in-flight moves?

(A) 0.0013 (B) 0.125
(C) 0.012 (D) 0.2311

$P(\bar{X} < 38)$

CLT → when distribution unknown

B

$P(\bar{X} < 38) = P\left(\frac{\bar{X} - 42}{8/6} < \frac{32 - 42}{8/6}\right)$

$= P(Z < -3) = 1 - P(Z < 3) = 1 - 0.9987$

$= 0.0013$

8. For a geometric distribution with $f(x) = \frac{1}{2^x}$, x = 1, 2, ..., using Chebyshev's inequality which of the following is correct?

(A) $P(|X + 2| \le 2) \ge \frac{1}{2}$

(B) $P(|X - 2| \le 2) \ge \frac{1}{2}$

(C) $P(|X - 2| > 2) \ge \frac{1}{2}$

(D) $P(|X - 2| < 2) \ge \frac{1}{2}$

$(1-x)^{-2}$

$E(x) = \sum_{1}^{\infty} x \cdot \frac{1}{2^x} = \frac{1}{2} + 2 \cdot \frac{1}{2^2} + 3 \cdot \frac{1}{2^3} + \dots$

$= \frac{1}{2}\left[1 + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{2^3} + \dots\right]$

$= \frac{1}{2}\left(1 - \frac{1}{2}\right)^{-2} = 2$

$E(x^2) = \sum x^2 \cdot \frac{1}{2^x} = \frac{1}{2} + 2^2 \cdot \frac{1}{2^2} + 3^2 \cdot \frac{1}{2^3} + \dots$

$= \frac{1}{2}\left[1 + 4 \cdot \frac{1}{2} + 9 \cdot \frac{1}{2^2} + \dots\right]$

$= \frac{1}{2}\left(1 + \frac{1}{2}\right)\left(1 - \frac{1}{2}\right)^{-3} = 6$

$V(x) = 6 - 2^2 = 2$

By Chebyshev's Inequality, $P(|x - \mu| \le t\sigma) \ge 1 - \frac{1}{t^2} = \frac{1}{2}$

$P(|x - 2| \le 2) \ge 1 - \frac{1}{\sqrt{2}^2}$

$t = \sqrt{2}$, $P(|x - 2| \le 2) \ge \frac{1}{2}$

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9. Find the least value of probability $P(1 \le X \le 7)$ where X is a random variable with $E(X) = 4$ and $V(X) = 4$.

(A) 1/9

(B) 2/9

(C) 4/9

(D) 5/9

CB Inequality $P(|x - \mu| \le k\sigma) \ge 1 - \frac{1}{k^2}$

$P(|x - 4| \le 2k) \ge 1 - \frac{1}{k^2}$

value of $(1 - 4 \le x - 4 \le 7 - 4)$

$P(|X-4| \leq 2k) \ldots$ $(1-4 \leq X-4 \leq 7-4)$

the least value of

$P(1 \leq X \leq 7) = P(1-4 \leq X-4 \leq 3)$

$P(-3 \leq X \leq 3)$

Put $2k = 3$, $k = 3/2$ $\frac{1}{k} = 2/3$

$P(1 \leq X \leq 7) = P(|X-4| \leq 3) \geq 1 - 4/9 = 5/9$

Hence, the least value is $5/9$

10. If $X \sim B(100, 0.5)$, using Chebyshev's Inequality obtain the lower bound for, $P(|X-50| < 7.5)$.

(A) 0.56 (B) 0.32

(C) 0.21 (D) 0.65

$\mu = np = 50$ $\sigma^2 = npq = 25$

$P(|X-\mu| \leq t\sigma) \geq 1 - \frac{1}{t^2}$

$P(|X-50| \leq 5t) \geq 1 - \frac{1}{t^2}$

Put, $5t = 7.5$, $t = 1.5$

$P(|X-50| < 5 \cdot 1.5) > 1 - \frac{1}{(1.5)^2}$

$P(|X-50| < 7.5) > 0.56$

Lower Bound is 0.56

Convert B to Z ...

11. The heights of 18-year-old men are approximately normally distributed with mean 68 inches and standard deviation 3 inches. What is the probability that a randomly selected 18-years-old man is between 67 and 69 inches tall.

(A) 0.2322 (B) 0.2365

(C) 0.2586 (D) 0.3212

X to Z $z = \frac{x - \mu}{\sigma} =$

$P(67 \leq X \leq 69)$

$= P(-0.33 \leq Z \leq 0.33)$

$= 0.6293 - 0.3707 = 0.2586$

12. Suppose that taxi and takeoff time for commercial jets is a random variable x with a mean of 8.5 minutes and a standard deviation of 2.5 minutes. What is the probability that for 36 jets on a given runway total taxi and takeoff time will be less than 320 minutes?

(A) 0.2315 (B) 0.8238

(C) 0.3256 (D) 0.3155

13. The Central Limit Theorem states that :

(A) if n is large then the distribution of the sample can be approximated closely by a normal curve

(B) if n is large, and if the population is normal, then the variance of the sample mean must be small

(C) if n is large, then the sampling distribution of the sample mean can be approximately closed by a normal curve

(D) if n is large, and if the population is normal, then the sampling distribution of the sample mean can be approximated closely by a normal curve

14. The central limit theorem tells us that the sampling distribution of the sample mean is approximately normal. Which of the following conditions are necessary for the theorem to be valid ?

(A) The sample size has to be large

(B) We have to be sampling from a normal population

(C) The population has to be symmetric

(D) Population variance has to be small

(D) . ' ˉ '

15. The Central Limit Theorem is important in Statistics because it allows us to use the normal distribution to make inferences concerning the population mean:

(A) provided that the population is normally distributed and the sample size is reasonably large

(B) provided that the population is normally distributed (for any sample size)

(C) provided that the sample size is reasonably large (for any sample size)

(D) None of these

16. The Central Limit Theorem is important in Statistics because :

(A) it tells us that large samples do not need to be selected

(B) it guarantees that, when it applies, the sample that are drawn are always randomly selected

(C) it enables reasonably accurate probabilities to be determined for events involving the sample average when the sample size is large regardless of the distribution of the variable

(D) it tells us that if several samples have product sample averages which seem to be different than expected, the next sample average will likely be close to its expected value its expected value

17. A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes.

(A) 19/20

(B) 19/23

(C) 19/24

(D) 24/19

18. Use Chebychev's inequality to determine how many times a fair coin must be tossed in order that the probability will be at least 0·90 that the ratio of the observed number of heads to the number of tosses will lie between 0·4 and 0·6.

(A) 300 (B) 250

(C) 245 (D) 255

19. Let $X_1, X_2, \ldots, X_n$ be i.i.d. variables with mean m and variance σ^2 and as $n \to \infty$,

$$(X_1^2 + X_2^2 + \ldots + X_n^2)/n \xrightarrow{p} c,$$

for some constant c; $(0 \le c \le \infty)$. Find c.

(A) $c = \sigma^2 + \mu^2$ (B) $c = \sigma^2$

(C) $c = \sigma^2 - \mu^2$ (D) $c = \mu^2$

20. How large a sample must be taken in order that the probability will be at least 0·95 that $\overline{X}_n$ will be lie within 0·5 of μ.μ is unknown and $\sigma = 1$.

(A) $n = 80$

(B) $n < 80$

(C) $n \geq 80$

(D) $n > 70$

(C) $n \geq 80$ (D) $n > 70$

1. For which of the following the law of large numbers can be applied to the independent variables $X_1, X_2, \ldots$, i.e., X_i's.

(A) if X_i assume that values i and –i with equal probabilities

(B) If X_i can have only two values with equal probabilities i^a and $-i^a$, if $\alpha < \frac{1}{2}$

(C) if X_i assume that values i and –i with unequal probabilities

(D) None of the above

(D) None of the above

22. Let $\{X_k\}$ be mutually independent and identically distributed random variables with mean m and finite variance. If $S_n = X_1 + X_2 + \dots + X_n$ then which of the following holds?

(A) the law of large numbers does not hold for the sequence $\{S_n\}$

(B) the law of large numbers holds for the sequence $\{S_n\}$

(C) it satisfies the central limit theorem

(D) None of the above

23. The sequence $\{X_k\}$ of independent random variables defined as follows:

$$P[X_k = \pm 2^k] = 2^{-(2k+1)}$$
$$P[X_k = 0] = 1 - 2^{-2k}$$

then which of the following statement holds?

(A) (Weak) Law of large numbers, does not holds for the sequence of independent r.v.'s $\{X_k\}$

(B) (Weak) Law of large numbers, holds for the sequence of independent r.v.'s $\{X_k\}$

(C) (Strong) of large numbers, holds for the sequence of independent r.v.'s $\{X_k\}$

(D) None of the above

24. Let $X_1, X_2, \ldots, X_n$ be jointly normal with $E(X_i) = 0$ and $E(X_i^2) = 1$ for all i and $Cov(X_i, X_j) = \rho$ if $|j - i| = 1$ and 0 otherwise, then which of the following statement holds ?

(A) WLLN does not hold for the sequence $|X_n|$

(B) SLLN holds for the sequence $|X_n|$

(C) WLLN holds for the sequence $|X_n|$

(D) None of the above