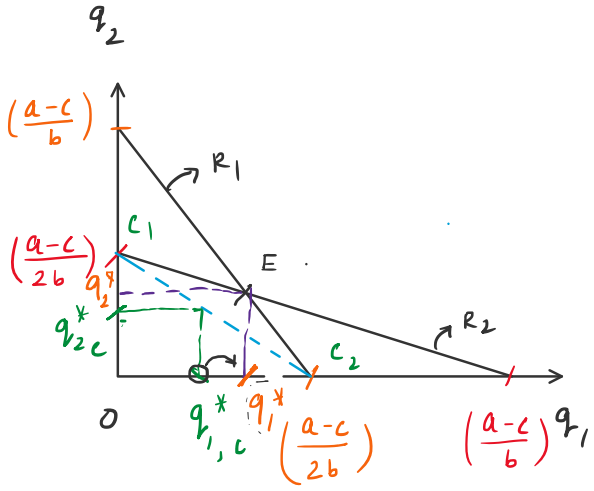
$$q_1 + q_2 = \left(\frac{a-c}{2b}\right)$$

Assuming that the firms will split the output equally, we get:-

$$q_{1c}^* = q_{2c}^* = \left(\frac{a-c}{4b}\right)$$



Note: Under Cournot Model:

R_1: $2bq_1 + bq_2 = (a-c)$ ... (i)

R_2: $bq_1 + 2bq_2 = (a-c)$ ... (ii)

Under Cartel:

JP: $q_1 + q_2 = \left(\frac{a-c}{2b}\right)$ ... (iii)

C_1C_2: Locus of output levels that both firms produce under cartel.

The cournot output level indicates the individual π-max output levels for each of the firms. If both the firms collude to max joint profits, then both individually produce less output as compared to the cournot setup. ∴ Less output ⇒ Less profit. Hence both the firms in the cartel will have an incentitive to cheat (to get higher individual profit). Hence a cartel is "unstable".

If: MC of the participating firms in cartel are different. then also cartel will be unstable.

Stakelberg's Model

There is one firm ⇒ Leader [He decides his output level first].
Other firm ⇒ Follower [He decides his optimal output, after the Leader's decision].

As the Leader decides on his optimal output level first, he can produce more and earn higher profits. This is the "First Mover Advantage".

8. Consider 2 firms participating the mkt. Mkt demand $P = a - bq$, $q = q_1 + q_2$. $MC_1(q_1) = MC_2(q_2) = c > 0$.
If Firm I is the leader. Find $q_1^*, q_2^*, \pi_1^*, \pi_2^*$.

For the follower: Firm II: it will observe the output level of the firm I & then decide on its optimal output level q_2^* (given by R_2).

For the leader: Firm I:
Firm I will first anticipate how firm II will react and incorporate this info into his optimal strategy to obtain q_1^*.

For Firm II: $\pi_2 = P \cdot q_2 - C_2 = [a - b(q_1 + q_2)] \cdot q_2 - c \cdot q_2$

$$\frac{\partial \pi_2}{\partial q_2} = 0 \Rightarrow a - b(q_1 + q_2) - bq_2 - c = 0$$

$$\Rightarrow bq_1 + 2bq_2 = (a - c).$$

$$\Rightarrow q_2 = \frac{(a-c) - bq_1}{2b} \quad \ldots\ldots (R_2).$$

Firm I will incorporate (R_2) to maximize π_1

... I will incorporate (R_2) to maximize π_1

∴ For Firm I : $\pi_1 = P.q_1 - C_1 = [a - b(q_1 + q_2)] . q_1 - c.q_1$

$$\pi_1 = a q_1 - b q_1^2 - b q_1 q_2 - c q_1 .$$

$$\pi_1 = (a-c) q_1 - b q_1^2 - b q_1 \left[\frac{(a-c) - b q_1}{2b}\right]$$

$$\pi_1 = (a-c) q_1 - b q_1^2 - \frac{(a-c)}{2} . q_1 + \frac{b}{2} q_1^2$$

$$\pi_1 = \left(\frac{a-c}{2}\right) q_1 - \frac{b}{2} q_1^2 .$$

$$\frac{d\pi_1}{dq_1} = 0 \Rightarrow \left(\frac{a-c}{2}\right) - b q_1 = 0 \Rightarrow q_1^* = \left(\frac{a-c}{2b}\right)$$

From R_2 : $q_2^* = \dfrac{(a-c) - b q_1^*}{2b} = \dfrac{(a-c) - b\left(\frac{a-c}{2b}\right)}{2b}$

$$= \frac{\frac{1}{2}(a-c)}{2b} = \left(\frac{a-c}{4b}\right) .$$

HW : $P^* = a - b(q_1^* + q_2^*)$

$\pi_1^* =$ $\pi_2^* =$

Check $\pi_1^* > \pi_2^*$

Q. Consider 3 firms in the mkt with the mkt demand curve $P = 1 - Q$, $Q = q_1 + q_2 + q_3$. If $MC = 0$ for all firms:

(i) Find the Cournot-Nash Equilibrium.

$\pi_1 = P.q_1 = (1 - q_1 - q_2 - q_3) . q_1$.

$\pi_2 = P.q_2 = (1 - q_1 - q_2 - q_3) . q_2$

$\pi_3 = P.q_3 = (1 - q_1 - q_2 - q_3) . q_3$.

$\pi_3 = P \cdot q_3 = (1 - q_1 - q_2 - q_3) \cdot q_3$.

Firm I : $\frac{\partial \pi_1}{\partial q_1} = 0 \Rightarrow 1 - 2q_1 - q_2 - q_3 = 0 \Rightarrow 1 = 2q_1 + q_2 + q_3$.

Firm II : $\frac{\partial \pi_2}{\partial q_2} = 0 \Rightarrow \cdots\cdots = 0 \Rightarrow 1 = q_1 + 2q_2 + q_3$

Firm III : $\frac{\partial \pi_3}{\partial q_3} = 0 \Rightarrow \cdots\cdots = 0 \Rightarrow 1 = q_1 + q_2 + 2q_3$.

As MC = 0 is same for all firms. $q_1^* = q_2^* = q_3^* = \frac{1}{4}$.

$p^* = 1 - q^* = 1 - \frac{3}{4} = \frac{1}{4}$.

$\pi_1^* = \pi_2^* = \pi_3^* = \frac{1}{16}$.

HW

(ii) If firms 2 & 3 collude. Find the Cournot Nash Equilibrium and compare the profit levels of the firms.