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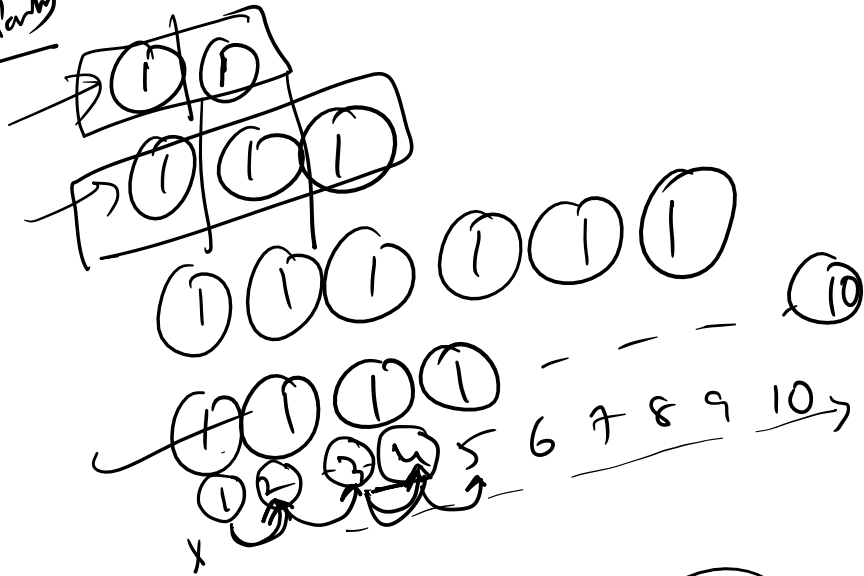
① $y = \alpha + \beta x_1$

② $y = \alpha + \beta_1 x_1 + \beta_2 x_2$

③ $y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$

④ $y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_{96} x_{96}$

New Year Party



* Linear dependence \rightarrow one is scalar multiple of other

vector space \rightarrow linearly dependent future

$V_3(\mathbb{R}) \rightarrow$ field of Real Numbers



$$\left\{ \begin{matrix} \alpha \\ \beta \\ \gamma \end{matrix} \right\} \rightarrow \left\{ (2,1,2), (8,4,8) \right\}$$

$$\begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix} =$$

$$\left\{ (1,2,0), (0,3,1), (-1,0,1) \right\}$$

$$\alpha(1,2,0) + \beta(0,3,1) + \gamma(-1,0,1) = (0,0,0)$$

$$\begin{cases} \alpha - \gamma = 0 \\ 2\alpha + 3\beta = 0 \\ \beta + \gamma = 0 \end{cases}$$

$$\boxed{\alpha = \beta = \gamma = 0}$$

Linearly independent

Basis of a vector space

Spanning set of all linearly independent vectors

Dimension \rightarrow No. of elements in any Basis of a finite d.V.S.

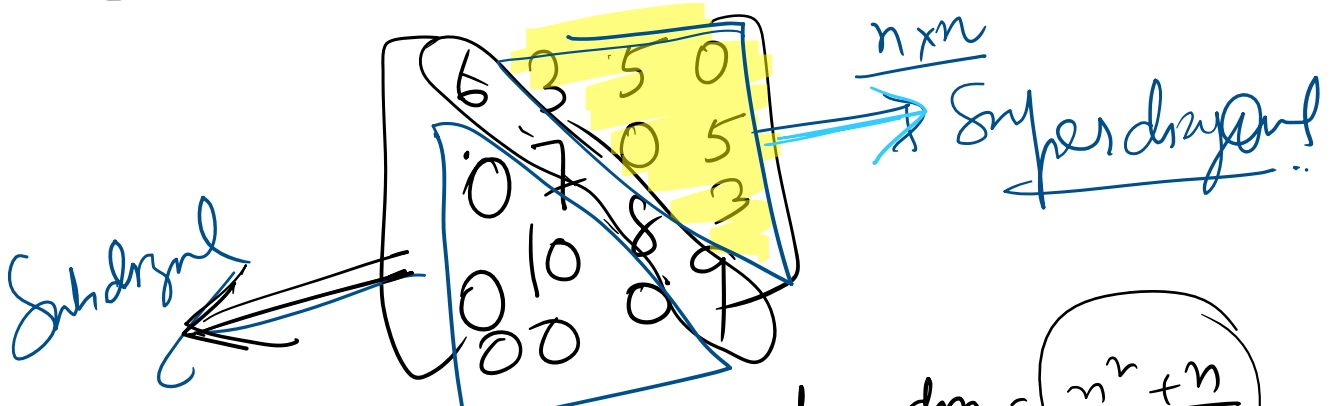
$$\dim(W_1 \cup W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2)$$

Some rules for the rank of a matrix

① Dimension of a vector space $\begin{matrix} k \times c \\ m \times n \text{ matrix} \end{matrix} \Rightarrow (mn - 1)$

- (i) Dimension of $n \times n$ matrix where $\sum_{row=1}^n \text{row} = 0 \Rightarrow (mn - 1)$
- (ii) where $\sum \text{each row} = 0 \Rightarrow mn - m$
- (iii) where $\sum \text{each column} = 0 \Rightarrow mn - n$
- (iv) where $\sum \text{each row \& column} = 0 \Rightarrow (mn - m - n + 1)$
- (v) $n \times n$ matrix all diagonal $\Rightarrow 0$

- (vi) $n \times n$ scalar matrices $\Rightarrow \text{dim} = n$
- (vii) $n \times n$ triangular matrix $\Rightarrow \text{dim} = 1$
- (viii) $n \times n$ Tri-diagonal matrix $\Rightarrow (3n - 2)$



- (ix) $n \times n$ symmetric matrix $\text{dim} = \frac{n^2 + n}{2}$
- (x) $n \times n$ skew symmetric matrix $\text{dim} = \frac{n^2 - n}{2}$
- (xi) Upper triangular / Lower triangular $\text{dim} = \frac{n^2 + n}{2}$
- (xii) dim (null matrix) $\Rightarrow 0$

(1) $\dim(\text{null matrix}) = 0$



$$w_1 = \begin{pmatrix} a & -a \\ c & d \end{pmatrix}$$

$$w_2 = \begin{pmatrix} a & -b \\ -a & d \end{pmatrix}$$

$m = \dim(w_1 \wedge w_2)$ then $(m, n) = ??$
 $n = \dim(w_1 + w_2)$

Netflix

\rightarrow dim

Apple TV

\rightarrow scalar

Members of the set

5 7

Hammer
 2-4
 5-8

$$w_1 + w_2 = \begin{pmatrix} 2a & -a-b \\ c-a & 2d \end{pmatrix} \Rightarrow 4$$

$$\begin{aligned} w_1 \wedge w_2 &= (w_1) + w_2 - (w_1, w_2) \\ &= (4-1) + (4-1) - 4 \\ &= 6 - 4 = 2 \end{aligned}$$

$$\left. \begin{aligned} m &= 2 \\ n &= 4 \end{aligned} \right\}$$

** $M_5(\mathbb{R})$ vector space 5x5
 \rightarrow all real entries

\dots $M_n(\mathbb{R})$ subspace of \dots

$W \subset M_5(\mathbb{R})$

Subspace of
Symmetric
matrices

find dimension of W .

~~Ans~~

Ans:

$$n=5$$

$$\dim = \frac{5 \times 5}{2}$$

$$\frac{n(n-1)}{2}$$

$$= \frac{5 \times 5}{2} = 10$$

Q which is a 2D subspace of \mathbb{R}^3 over \mathbb{R} .

X a) $\{(0, x, 0) \mid x \in \mathbb{R}\}$

X b) $\{(0, x, 0) \mid x \in \mathbb{R}\} \cup \{(0, 0, y) \mid y \in \mathbb{R}\}$

X c) $\{(x, y, 0) \mid x, y \in \mathbb{R} \text{ \& } x+y=0\} \rightarrow y = -x$
 $\{x, -x, 0\} \rightarrow \text{1D}$

d) $\{(0, x, z) \mid x, z \in \mathbb{R}\}$

$(0, 1, 0) \in W_2$ + $(0, 0, 1) \in W_2 \rightarrow (0, 1, 1) \notin W_2$

1, 2, 3, 4, ... $U_n = n$
2, 4, 6, ... $U_n = 2n$
3, 6, 9, ... $U_n = 3n$

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2017

6x6 \rightarrow subspace of Symmetric

$6 \times 6 \rightarrow$ symmetric $\left(\frac{6 \times 7}{2} \right)$
 $\frac{n(n+1)}{2} \Rightarrow \frac{6 \times 7}{2} = 21$

Dimension of Subspace $\left\{ (x_1, x_2, x_3, x_4, x_5) \right\}$
 $\{ 3x_1 - x_2 + x_3 = 0 \}$ of \mathbb{R}^5

$1 \ 2 \ 3 \ 4$

$3x_1 - x_2 + x_3 = 0$
 $x_2 = 3x_1 + x_3$

Any $(x_1, x_2, \dots, x_5) \in W$ can be written as

$(x_1, 3x_1 + x_3, x_3, x_4, x_5) = x_1(1, 3, 0, 0, 0) + x_3(0, 1, 1, 0, 0)$
 $+ x_4(0, 0, 0, 1, 0) + x_5(0, 0, 0, 0, 1)$

These 4 span W & W is linearly independent. $\rightarrow 4$

$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ x & y & z \end{bmatrix}$

$V = \left\{ (x, y, z) \in \mathbb{R}^3; \det(A) = 0 \right\}$

$\frac{0}{1/9/3}$

$A = \begin{bmatrix} x & y & z \end{bmatrix}$
Then $\dim(V) = ?$

$$\begin{array}{|c|c|c|c|} \hline 0 & 1 & 2 & 3 \\ \hline \end{array}$$

$x + y = z$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ x & y & z \end{vmatrix} = 2z + 2y + 3x - 2x - 3y - 2z = x - y$$

$$V = \{ (x, y, z) \in \mathbb{R}^3 ; \det(A) = 0 \}$$

$x = y$

$$V = \{ x, x, z \}$$

$\dim(V) = 2$