07 July 2023 07:54 PM

(1) Classical definition of probability of an event A is given by,

$$P(A) = \frac{\text{no. of events favourable to A}}{\text{tot no. of elementary events}}$$

$$P(A) = \frac{m(A)}{n(A)}$$

(2) Total probability theorem in case 2 events (say A_1 and A_2)
[A_1 and A_2 are mutually exclusive]

$$P[A_1 \cup A_2] = P(A_1) + P(A_2) - \underbrace{P(A_1 \cap A_2)}_{\to 0}$$

$$P(A_1 \cup A_2) = P(A_1) + P(A_2)$$

if we have $A_1, A_2, A_3, \ldots, A_n$.

then $$P\left(\bigcup_{i=1}^{n} A_i\right) = P(A_1) + P(A_2) + \cdots + P(A_n)$$

$$P(A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n) = \sum_{i=1}^{n} P(A_i)$$

(3) If the events A_1 and A_2 are mutually exclusive as well as exhaustive then

$$P(A_1 \cup A_2) = 1$$ (sure event)

(total probability

$P(A_1 \cup A_2) = 1$ (sure event)

$P(A_1) + P(A_2) = 1$ (total probability 1)

$P(A_1) = 1 - P(A_2)$

$P(A_2) = 1 - P(A_1)$

Q) Given $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(AB) = \frac{1}{4}$

(i) Find: $P(\bar{A})$, $P(A+B)$, $P(A/B)$, $P(\bar{A}B)$, $P(\bar{A}\bar{B})$, $P(\bar{A}+B)$

$P(A \cap B)^c = 1 - P(A \cup B)$

Solve:

$P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$

$P(A+B) = P(A) + P(B) - P(AB) = \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{7}{12}$

$P(A|B) = \frac{P(AB)}{P(B)} = \frac{1/4}{1/3} = 3/4$

$P(\bar{A}B) = P(B) - P(AB) = \frac{1}{3} - \frac{1}{4} = 1/12$

$P(\bar{A}\bar{B}) = 1 - P(A+B) = 1 - 7/12 = 5/12.$

$P(\bar{A}+B) = P(\bar{A}) + P(B) - P(\bar{A}B)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{12} = \frac{3}{4}$

4) Conditional probability

A given B $\Rightarrow$ $P(A/B) = \frac{P(AB)}{P(B)}$

$$A \text{ given } B \Rightarrow P(A/B) = \frac{P(AB)}{P(B)}$$

$$B \text{ given } A \Rightarrow P(B/A) = \frac{P(AB)}{P(A)}$$

Now, $P(AB) = P(A/B) \cdot P(B)$
$P(AB) = P(B/A) \cdot P(A)$ } compound probability. (2 events)

More than two events : $[A_1, A_2, A_3, \ldots, A_n]$

2 events : $P(A_1 \cap A_2) = P(A_1) \cdot P(A_2/A_1)$

3 " : $P(A_1 \cap A_2 \cap A_3) = P(A_1) P(A_2 \cap A_3 / A_1)$
$= P(A_1) P(A_2/A_1) P(A_3/A_1 \cap A_2)$

4 " : $P(A_1 \cap A_2 \cap A_3 \cap A_4) = P(A_1) P(A_2/A_1) P(A_3/A_1 \cap A_2) P(A_4/A_1 \cap A_2 \cap A_3)$

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n events : $P\left(\bigcap_{i=1}^{n} A_i\right) = P(A_1) P(A_2/A_1) P(A_3/A_1 \cap A_2) \ldots \ldots P\left(A_n / \bigcap_{i=1}^{n-1} A_i\right)$

(5) Let A_1 and A_2 are two independent events

then $P(A_1/A_2) = \frac{P(A_1 \cap A_2)}{P(A_2)}$

$= \frac{P(A_1) \cdot \cancel{P(A_2)}}{\cancel{P(A_2)}}$

$\therefore P(A_1/A_2) = P(A_1)$

(6) If A_1 & A_2 are indp, $\left[P(A_1 \cap A_2) = P(A_1) P(A_2) \right]$

i.e. $P(A_2 | A_1) = P(A_2) \longrightarrow (1)$

$\therefore P(A_1 \cap A_2) = P(A_1) \cdot P(A_2 / A_1) = P(A_1) P(A_2)$

(Proved).

(7) Suppose 2 events A_1 and A_2 are statistically independent, then show/prove that. $\hookrightarrow P(A_1 \cap A_2) = P(A_1) P(A_2)$

a) A_1^c & A_2 are indp $\Rightarrow P(A_1^c \cap A_2) = P(A_1^c) P(A_1)$

b) A_1 & A_2^c are indp $\Rightarrow P(A_1 \cap A_2^c) = P(A_1) P(A_2^c)$

c) A_1^c & A_2^c are indp. $\Rightarrow P(A_1^c \cap A_2^c) = P(A_1^c) P(A_2^c)$

a) if A_1 & A_2 are indp, $P(A_1 \cap A_2) = P(A_1) \cdot P(A_2) \longrightarrow (1)$

A_1 and A_1^c are mutually exclusive events and so A_2 can occur in 2 mutually exclusive way

$$A_2 = (A_1 \cap A_2) \cup (A_1^c \cap A_2)$$

$$P(A_2) = P\left[(A_1 \cap A_2) \cup (A_1^c \cap A_2) \right]$$

$$P(A_2) = P(A_1 \cap A_2) + P(A_1^c \cap A_2)$$

$$P(A_2) = P(A_1 \cap A_2) + P(A_1^c \cap A_2)$$

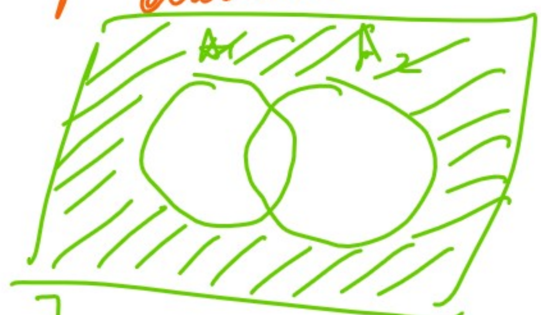
$$P(A_2) = P(A_1) \cdot P(A_2) + P(A_1^c \cap A_2)$$

$$P(A_1^c \cap A_2) = P(A_2) - P(A_1)\,P(A_2)$$

$$P(A_1^c \cap A_2) = P(A_2)\,[1 - P(A_1)]$$

$$P(A_1^c \cap A_2) = P(A_2)\,P(A_1^c)$$ → A_1^c and A_2 are independent events.

(c) $P(A_1^c \cap A_2^c) = P(A_1 \cup A_2)^c$



or, $P(A_1^c \cap A_2^c) = 1 - P(A_1 \cup A_2)$

$$= \; - [P(A_1) + P(A_2) - P(A_1 A_2)]$$

$$= 1 - [P(A_1) + P(A_2) - P(A_1)\,P(A_2)]$$

$$P(A_1^c \cap A_2^c) = 1 - P(A_1) - P(A_2) + P(A_1)\,P(A_2)$$

$$= \underbrace{(1 - P(A_1))} - P(A_2)\,[\underbrace{1 - P(A_1)}]$$

$$= \underbrace{(1 - P(A_1))}\,(1 - P(A_2))$$

$$P(A_1^c \cap A_2^c) = P(A_1^c)\;P(A_2^c)$$