Application of differentials

19 August 2023 08:20

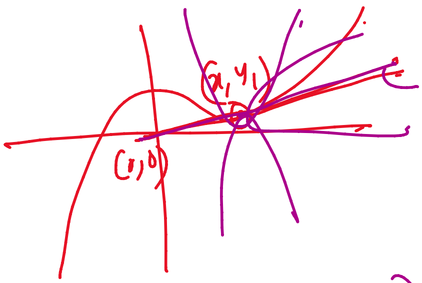
the tangent at the point (x_1, y_1) on the curve $y = x^3 + 3x^2 + 5$ passes through the origin, then (x_1, y_1) does NOT lie on the curve :

(A) $x^2 + \frac{y^2}{81} = 2$ (B) $\frac{y^2}{9} - x^2 = 8$

(C) $y = 4x^2 + 5$ (D) $\frac{x}{3} - y^2 = 2$

$1+1=2$ ✓ $9-1=8$ ✓

$9=4+5$ ✓ $\frac{1}{3}-81 \neq 2$ ✓



slope $= \left(\frac{dy}{dx}\right)_{x_1,y_1}$

Solve for x_1, y_1 and check the options

$\frac{dy}{dx} = 3x^2+6x$

$y_1 = x_1^3+3x_1^2+5$ — ①

$\left(\frac{dy}{dx}\right)_{x_1,y_1} = 3x_1^2+6x_1$

$y-0 = m(x-0)$

$y = mx$

$y = (3x_1^2+6x_1)x$

$y_1 = (3x_1^2+6x_1)x_1$ — ②

$3x_1^3+6x_1^2 = x_1^3+3x_1^2+5$

$2x_1^3+3x_1^2-5=0$ $x_1=1$ $y_1=9$

$2x_1^3-2x_1^2+5x_1^2-5=0$

$2x_1^2(x_1-1)+5(x_1^2-1)=0$

$2x_1^2(x_1-1)+5(x_1-1)(x_1+1)=0$

$(x_1-1)(2x_1^2+5x_1+5)=0$

$D = 5^2-4\times2\times5 < 0$

The sum of absolute maximum and absolute minimum values of the function

$f(x) = |2x^2 + 3x - 2| + \sin x \cos x$ in the interval

[0, 1] is :

(A) $3 + \frac{\sin(1)\cos^2(\frac{1}{2})}{2}$ (B) $3 + \frac{1}{2}(1 + 2\cos(1))\sin(1)$

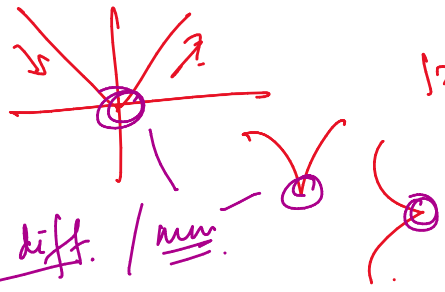
(C) $5 + \frac{1}{2}(\sin(1) + \sin(2))$ (D) $2 + \sin\left(\frac{1}{2}\right)\cos\left(\frac{1}{2}\right)$

$2x^2 + 4x - x - 2$
$= 2x(x+2) - 1(x+2)$
$(2x-1)(x+2)$

| | + trig.

$f(x) = |(2x-1)(x+2)| + \frac{\sin 2x}{2}$

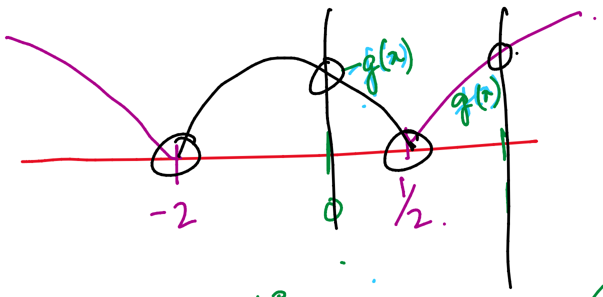
$g(x)$



$|x|$

not diff. / min.

$(2x-1)(x+2)$



$1 \sim \frac{\pi}{3} \sim 60°$
$\cos 1 \sim \frac{1}{2}$

$0 < x \le \frac{1}{2}$

$f(x) = -2x^2 - 3x + 2 + \frac{\sin 2x}{2}$

$f'(x) = -4x - 3 + \frac{1}{2} \cdot 2\cos 2x$

decreasing

$x = 0$ $f'(x) = -2 < 0$. $x = \frac{1}{2}$ $f'(x) = -5 + \cos 1 < 0$.

$\frac{1}{2} < x \le 1$

$f(x) = 2x^2 + 3x - 2 + \frac{\sin 2x}{2}$

increasing

$f'(x) = 4x + 3 + \cos 2x$.

$f'(x)$ $x = \frac{1}{2}$ $5 + \cos 1 > 0$ $x = 1$ $7 - \frac{1}{2} = 6.5 > 0$

$f(x)_{min} = \frac{\sin 1}{2}$

$f(1) = 3 + \frac{\sin 2}{2}$ $f(0) = +2$

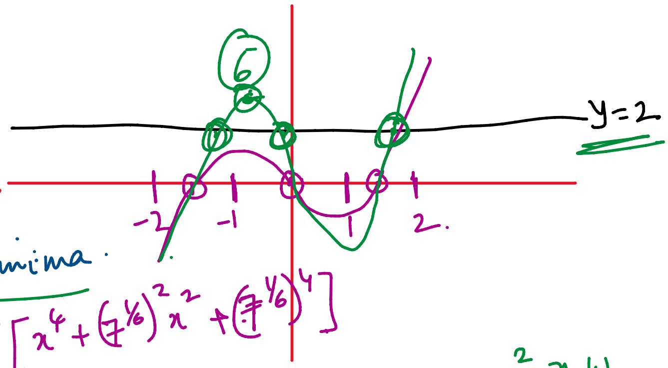
$3 + \frac{1}{2}\sin 1(1 + 2\cos 1)$

$f(x)_{max} + f(x)_{min} = 3 + \frac{1}{2}(\sin 1 + \sin 2)$

$= 3 + \frac{1}{2}[\sin 1 + 2\cos 1 \sin 1]$

The number of distinct real roots of the equation $x^7 - 7x - 2 = 0$ is

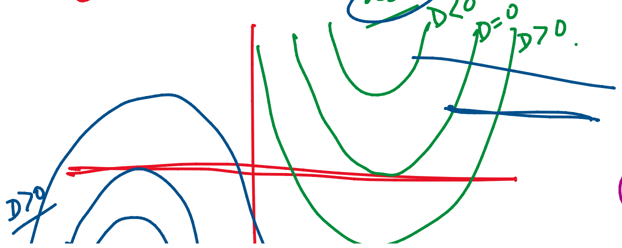
(A) 5 (B) 7 (C) 1 (D) 3

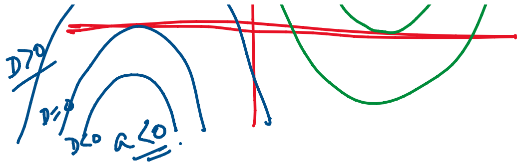


Let λ^* be the largest value of λ for which the function $f_\lambda(x) = 4\lambda x^3 - 36\lambda x^2 + 36x + 48$ is increasing for all $x \in R$. Then $f_{\lambda^*}(1) + f_{\lambda^*}(-1)$ is equal to :

(A) 36 (B) 48

(C) 64 (D) 72





$\lambda^* = \frac{1}{3}.$

$\lambda > 0 \quad D \le 0$

$36\lambda^2 - 12\lambda \le 0$

$\lambda(3\lambda - 1) \le 0$

$0 \le \lambda \le 1/3$

Let f (x) be a polynomial function such that f (x) + f ' (x) + f ''(x) = x^5 + 64. Then, the value of $\lim_{x\to 1} \frac{f(x)}{x-1}$

(A) – 15 (B) – 60

(C) 60 (D) 15

Let $f : R \rightarrow R$ and $g : R \rightarrow R$ be two functions defined by $f(x) = \log_e(x^2 + 1) - e^{-x} + 1$ and $g(x)=\frac{1-2e^{2x}}{e^x}$. Then, for which of the following range of α, the inequality

$$f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right) > f\left(g\left(\alpha-\frac{5}{3}\right)\right)$$ holds?

(A) (2, 3)
(B) (–2, – 1)
(C) (1, 2)
(D) (–1, 1)

Water is being filled at the rate of $1\ \text{cm}^3/\sec$ in a right circular conical vessel (vertex downwards) of height 35 cm and diameter 14 cm. When the height of the water level is 10 cm, the rate (in $\text{cm}^2/\sec$) at which the wet conical surface area of the vessel increases is

(A) 5

(B) $\frac{\sqrt{21}}{5}$

(C) $\frac{\sqrt{26}}{5}$

(D) $\frac{\sqrt{26}}{10}$

If the angle made by the tangent at the point (x_0, y_0) on the curve $x = 12(t + \sin t \cos t)$,

$y = 12(1 + \sin t)^2, 0 < t < \frac{\pi}{2}$, with the positive x-axis

is $\frac{\pi}{3}$, then y_0 is equal to

(A) $6\left(3+2\sqrt{2}\right)$ (B) $3\left(7+4\sqrt{3}\right)$

(C) 27 (D) 48

Let $f(x)=\left|(x-1)(x^2-2x-3)\right|+x-3,\ x\in R$. If m and M are respectively the number of points of local minimum and local maximum of f in the interval (0, 4), then m + M is equal to ____

The sum of the absolute minimum and the absolute maximum values of the function $f(x) = |3x - x^2 + 2| - x$ in the interval $[-1, 2]$ is :

(A) $\frac{\sqrt{17}+3}{2}$

(B) $\frac{\sqrt{17}+5}{2}$

(C) 5

(D) $\frac{9-\sqrt{17}}{2}$

Let S be the set of all the natural numbers, for which the line $\frac{x}{a}+\frac{y}{b}=2$ is a tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at the point (a, b), $ab \neq 0$. Then:

(A) $S = \phi$ (B) $n(S) = 1$

(C) $S = \{2k : k \in N\}$ (D) $S = N$

Let $f(x) = 2\cos^{-1}x + 4\cot^{-1}x - 3x^2 - 2x + 10$, $x \in [-1, 1]$. If $[a, b]$ is the range of the function then $4a - b$ is equal to:

(A) 11 (B) $11 - \pi$ (C) $11 + \pi$ (D) $15 - \pi$

Consider a cuboid of sides 2x, 4x and 5x and a closed hemisphere of radius r. If the sum of their surface areas is a constant k, then the ratio x : r, for which the sum of their volumes is maximum, is :

(A) 2 : 5 (B) 19:45 (C) 3 : 8 (D) 19 : 15

If y = y(x) is the solution of the differential equation $x\frac{dy}{dx}+2y=xe^x, y(1)=0$, then the local maximum value of the function $z(x) = x^2y(x)-e^x$, $x \in R$ is :

(A) $1 - e$ (B) 0 (C) $\frac{1}{2}$ (D) $\frac{4}{e}-e$