28 December 2023 16:56

$f(x) + 2f(x-4) = x.$

$f(x)$ $f(x-4)$ Method of degrees.

degree = 1 $\boxed{f(x) + 2f(x-4)} = x$ degree = 1

degree = 1

let $\boxed{f(x) = ax + b}$.

$\therefore f(x-4) = a(x-4) + b$

$\therefore f(x-4) = ax - 4a + b.$

$\boxed{2f(x-4) = 2ax - 8a + 2b}.$

$f(x) + 2f(x-4) = 3ax + 3b - 8a.$

$x = 3ax + (3b - 8a)$

equate the coefficients

$3a = 1 \Rightarrow \boxed{a = \frac{1}{3}}$

$3b - 8a = 0$

$\boxed{b = \frac{8}{3}a = \frac{8}{9}}$

$f(x) = \frac{x}{3} + \frac{8}{9} = \frac{3x+8}{9}$ ✓

$f(x-4) = \frac{3(x-4)+8}{9} = \frac{3x-4}{9}$

$2f(x-4) = \frac{6x-8}{9}$ ✓

$f(x) + 2f(x-4) = \frac{9x}{9} = x.$

✓ $f(x) = x - x^3$ ✓

$f(x-4) = (x-4) - (x-4)^3$

✓ $2f(x-4) = 2(x-4) - 2(x-4)^3.$

$f(x) f(x-4) = -3x^3 \ldots$

$f(x) + f(x-4) = x.$

$f(x) = ax + b.$ $f(x-4) = a(x-4) + b = ax - 4a + b.$

$f(x) + f(x-4) = 2ax + 2b - 4a.$

$x = 2ax + 2b - 4a.$

$\boxed{a = \frac{1}{2}}$ $\boxed{b = 2a = 1}$

$f(x) = \frac{1}{2}x + 1$

degree n degree n

$f(x) + f(x-4) = x^2.$

$f(x) = ax^2 + bx + c.$

$\boxed{f(x) + 2f(\frac{1}{x})} = x.$ degree = 1

degree = 1

$f(x) = x^n.$

$f(\frac{1}{x}) = x^{-n}.$

$f(x) = ax + bx^{-1} + c.$

$y = x + \frac{1}{x}$. $y = x - \frac{1}{x}$.

$\lim_{x\to 0} (1+x)^{\frac{1}{x}}$.

If you have $f(x)$ in the power $\to$ always take log on both sides.

$y = (1+x)^{1/x}$. $\lim_{x\to 0} y = ?$ e.

$\log a^b = b \log a$

$\log_e y = \frac{1}{x} \log_e(1+x)$

$\lim_{x\to 0} \log_e y = \lim_{x\to 0} \frac{\log(1+x)}{x}$.

$\log_e\left(\lim_{x\to 0} y\right) = \lim_{x\to 0} \frac{\frac{1}{1+x}}{1} = 1$

$\lim_{x\to 0} y = e^1 = e$.

$\lim_{x\to 0} (x^2 + x + 1)^{\frac{1}{x^3}} = \lim_{x\to 0} \left\{1 + (x^2+x)\right\}^{\frac{1}{x^3}}$ $\to 0$. $\to \infty$.

$y = \left(x + \frac{1}{x}\right)$

$\left(x+\frac{1}{x}\right)^2 - 4\left(x+\frac{1}{x}\right) + 3 = 0$.   find x

$x + \frac{1}{x} = a$

$a^2 - 4a + 3 = 0$.

$(a-3)(a-1) = 0$.

$\boxed{a = 3, 1}$

$x + \frac{1}{x} = 3, 1$

$x^2 + 1 = 3x, x$.

$D = 5 \to 2$ sols

$x^2 + 1 - 3x = 0$

$x^2 + 1 - x = 0$.

$D = -3$ ✗

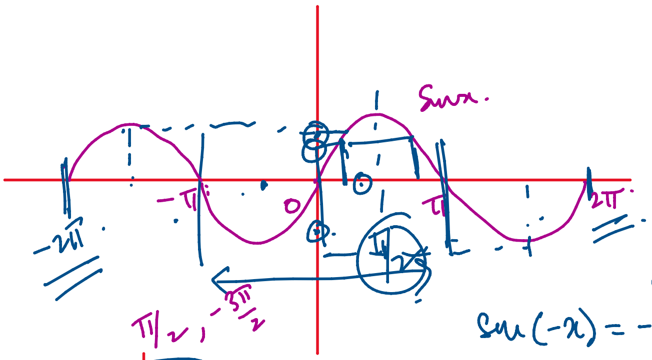
$y = \sin x + \frac{1}{\sin x}$.

$\left(\sin x + \frac{1}{\sin x}\right)^2 - 4\left(\sin x + \frac{1}{\sin x}\right) + 3 = 0$

$\sin^2 x - 3\sin x + 1 = 0$ $\sin^2 x - \sin x + 1 = 0$ ✗

$\sin x = \frac{3 \pm \sqrt{5}}{2}$. $\sqrt{5} = 2.2$

$\frac{3 \pm 2.2}{2} = \cancel{2.6}, 0.4$



$y = \sin x$

$\boxed{y' = \cos x}$

$\Downarrow$

even

$y = \cos x$

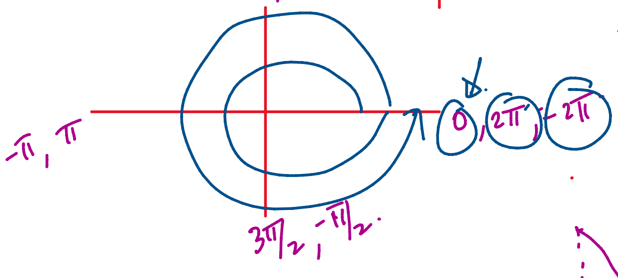
$\boxed{y' = -\sin x}$

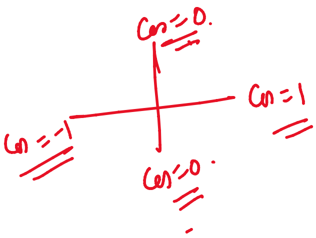
$-2\pi \le x \le 2\pi$

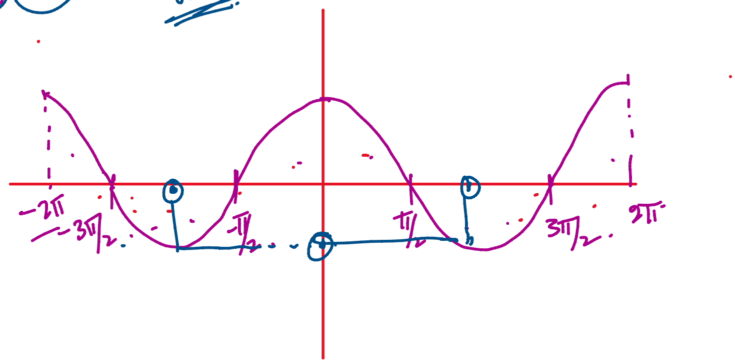
$\sin(-x) = -\sin x$

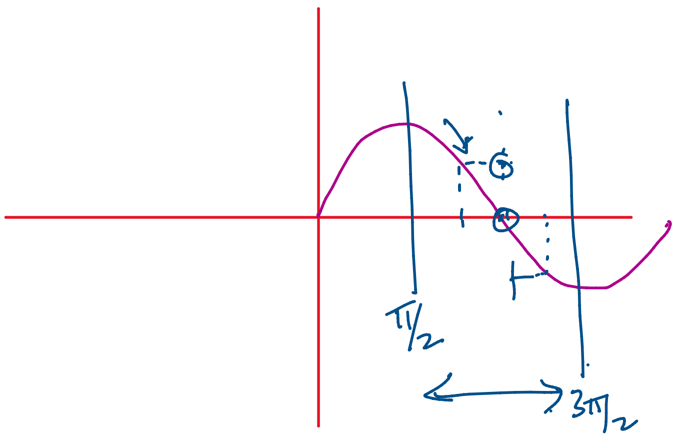
$\Downarrow$

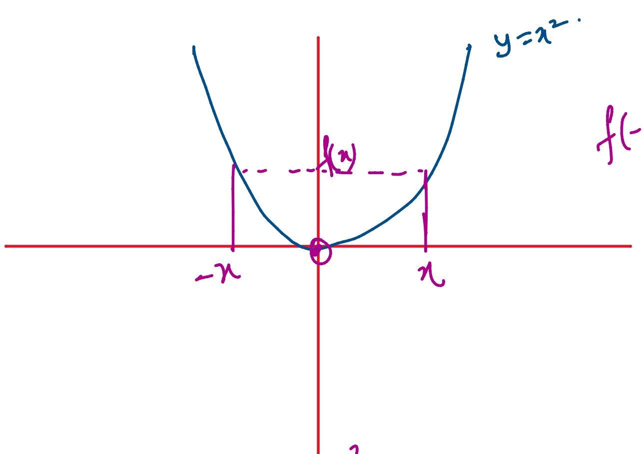
odd







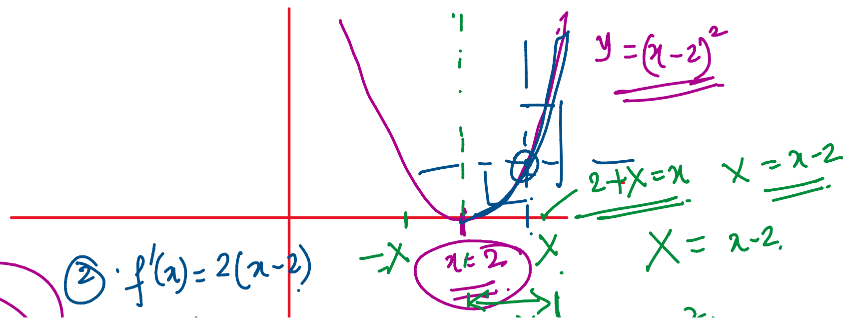




$y' = 2x$ ✓ even

$f(-x) = f(x)$ ✓ even

$f(x) = (x-2)^2$



$2 + X = x$ $\quad X = x-2$

$X = x-2$

① $f(-x) = (x+2)^2$

② $f'(x) = 2(x-2)$

① $f(-x) = (x+2)^2$.

$f(-x) = f(x)$ X

$f(-x) = -f(x)$ X

$x=0$ is the central axis

② $f'(x) = 2(x-2)$

$\Downarrow$

even

$-X$ $x=2$ X

$X=0$ X

$X = x-2$.

$y = X^2$.

$f(-X) = f(X)$

$f[-(x-2)] = f(x-2)$