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(i) $C_1 + 2C_2 + 3C_3 + 4C_4 + \cdots + nC_n$.

$C_1 + C_2 + C_2 + \cdots + C_n = 2^n - 1$

${}^nC_r$.

Method 1

$T_r = rC_r = r\,{}^nC_r = r \cdot \frac{n!}{r!(n-r)!}$

$r! = r \cdot (r-1)!$

$T_r = \frac{\cancel{r}\, n!}{\cancel{r} \cdot (r-1)!(n-r)!} = \frac{n!}{(r-1)!(n-r)!}$

$5! = 5 \cdot 4!$

$(1\times2\times3\times4)\times5$

$4!$

$T_r = \frac{n(n-1)!}{(r-1)!(n-r)!} = n\,{}^{n-1}C_{r-1}$

$\sum T_r = \sum_{r=1}^{n} n\,{}^{n-1}C_{r-1} = n\,{}^{n-1}C_0 + n\,{}^{n-1}C_1 + n\,{}^{n-1}C_2 + \cdots + n \cdot {}^{n-1}C_{n-1}$

$= n\left[{}^{n-1}C_0 + {}^{n-1}C_1 + {}^{n-1}C_2 + \cdots + {}^{n-1}C_{n-1}\right]$

$= n \cdot 2^{n-1}$

Method 2

$(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots + C_nx^n$.

differentiate both sides wrt x.

$n(1+x)^{n-1} = 0 + C_1 + 2C_2x + 3C_3x^2 + \cdots + nC_nx^{n-1}$

put $x=1$ $\quad n \cdot 2^{n-1} = C_1 + 2C_2 + 3C_3 + \cdots + nC_n$.

$2C_2 + 6C_3 + 12C_4 + \cdots + n(n-1)C_n = ?$

Method 1

$T_r = r(r-1)\,{}^nC_r = r(r-1)\frac{n!}{r!(n-r)!}$

$= \cancel{r}(\cancel{r-1})\frac{n!}{\cancel{r}(\cancel{r-1})(r-2)!(n-r)!}$

$= n(n-1)\boxed{\frac{(n-2)!}{(r-2)!(n-r)!}} \rightarrow {}^{n-2}C_{r-2}$

$= n(n-1)\,{}^{n-2}C_{r-2}$.

$$= n(n-1)\ {}^{n-2}C_{r-2}.$$

$$\sum T_r = n(n-1)\sum_{r=2}^{n} {}^{n-2}C_{r-2} = n(n-1)\cdot 2^{n-2}.$$

Method 2.

$$(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + C_4x^4 + \cdots + C_nx^n.$$

$$n(1+x)^{n-1} = C_1 + 2C_2x + 3C_3x^2 + 4C_4x^3 + \cdots + nC_nx^{n-1}$$

$$n(n-1)(1+x)^{n-2} = 2C_2 + 6C_3x + 12C_4x^2 + \cdots + n(n-1)C_nx^{n-2}$$

put $x=1$. $\quad n(n-1)\cdot 2^{n-2} = \underline{\text{Sum}}.$

(2). $$\overset{1}{C_0} + \overset{2}{\frac{C_1}{2}} + \overset{3}{\frac{C_2}{3}} + \cdots + \overset{n+1}{\frac{C_n}{n+1}} = ?$$

Method 1.

$$T_{r+1} = \frac{1}{r+1}\cdot C_r = \frac{1}{r+1}\cdot\frac{n!}{r!\,(n-r)!} = \frac{n!\,(n+1)}{(r+1)!\,(n-r)!\,(n+1)}$$

$$= \boxed{\frac{(n+1)!}{(r+1)!\,(n-r)!}}\cdot\frac{1}{n+1}$$

${}^{n+1}C_{r+1}$

$$\sum T_{r+1} = \frac{1}{n+1}\sum_{r=-1}^{n} {}^{n+1}C_{r+1} = \frac{1}{n+1}2^{n+1}$$

Method 2.

$$(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots + C_nx^n.$$

$$\int(1+x)^n dx = \int C_0\,dx + \int C_1x\,dx + \int C_2x^2dx + \cdots + \int C_nx^n dx.$$

$$\frac{(1+x)^{n+1}}{n+1} = C_0x + \frac{C_1x^2}{2} + \frac{C_2x^3}{3} + \cdots + \frac{C_nx^{n+1}}{n+1}$$

put $x=1$ $\quad \frac{2^{n+1}}{n+1} = \underline{\underline{\text{Sum}}}.$

$$C_0C_2 + C_1C_3 + C_2C_4 + \cdots + C_{n-2}C_n = ?$$

$$(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots + C_nx^n.$$

$C_0C_2 + C_1C_3 + C_2C_4 + \cdots + C_{n-2}C_n$.

$(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots + C_nx^n$.

$(x+1)^n = C_0x^n + C_1x^{n-1} + C_2x^{n-2} + \cdots + C_n$.

Multiply both sides and equate the coefficients of x^{n-2}.

$${}^{2n}C_{n-2} = C_0C_2 + C_1C_3 + C_2C_4 + \cdots + C_{n-2}C_n.$$

$$= \frac{(2n)!}{(n-2)!\,(n+2)!}.$$

If $\frac{1}{n+1}{}^nC_n + \frac{1}{n}{}^nC_{n-1} + \ldots + \frac{1}{2}{}^nC_1 + {}^nC_0 = \frac{1023}{10}$ then n is equal to :

A 9 ✓

B 6

C 7

D 8

$$\frac{2^{n+1}-1}{n+1}$$

Question 5: The positive integer just greater than $(1 + 0.0001)^{10000}$ is

(a) 4 (b) 5 (c) 2 (d) 3

$$\left(1+\frac{1}{n}\right)^n = 1 + n\cdot\frac{1}{n} + \frac{\cancel{n}(n-1)}{2!}\cdot\frac{1}{\cancel{n^2}} + \cdots + \frac{1}{n!}$$

$$= 2 + \left[\frac{(n-1)}{2!\,n} + \cdots + \frac{1}{n!}\right]$$

$e = 2.7$

$(e-2)$

$$\frac{(n-1)}{2!\,n} \qquad \frac{(n-1)(n-2)}{3!\,n^2} \qquad \frac{(n-1)(n-2)(n-3)}{4!\,n^3}$$

$$\frac{(1-1/n)(1-2/n)(1-3/n)}{4!}$$

$n \to \infty$. $\frac{1}{}$

$n \to \infty$. $\frac{1}{4!}$ $\leftarrow$ $\frac{\quad}{4!}$

$S = \frac{1}{2!} + \frac{1}{3!} + \cdots$

$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$

$e - 2 = \frac{1}{2!} + \frac{1}{3!} + \cdots$

Method 2

$\left(1+\frac{1}{n}\right)^n = x$.

$\log_e\left(1+\frac{1}{n}\right)^n = \log_e x$.

$n \log\left(1+\frac{1}{n}\right) = \log x$.

$\frac{\log(1+1/n)}{1/n} = \log_e x$.

$\lim_{n\to\infty} \frac{\log(1+1/n)}{1/n} = \lim_{n\to\infty} \log_e x$.

$\frac{1}{n} = p$. $n \to \infty$, $p \to 0$,

$\lim_{p\to 0} \frac{\log(1+p)}{p} = \log_e \lim_{n\to\infty} x$.

$\lim_{p\to 0} \frac{\frac{1}{1+p}}{1} = \log_e \lim_{n\to\infty} x$

$\log_e \lim_{n\to\infty} x = 1$

$\boxed{\lim_{n\to\infty} x = e}$

Let $(a + bx + cx^2)^{10} = \sum_{i=0}^{20} p_i x^i, a, b, c \in \mathbb{N}$.

$p, q, r \geq 0$

If $p_1 = 20$ and $p_2 = 210$, then $2(a + b + c)$ is equal to :

A 15

$q + 2r = 1$

General term

$= \frac{n!}{p!q!r!} a^p (bx)^q (cx^2)^r$.

$= \frac{10!}{\quad} a^p b^q c^r x^{q+2r}$.

A 15

B 8

C 6

D 12

$$= \boxed{\frac{10!}{p!\,q!\,r!} a^p b^q c^r x^{q+2r}}$$

$$\boxed{p+q+r = 10}$$

$q+2r = 1$

$q = 1 \quad p = 9$

$r = 0$

$\frac{10!}{9!\,1!\,0!} a^9 b = 20$

$10 a^9 b = 20$

$$\boxed{a^9 b = 2}$$

$q+2r = 2$

$q =$	0	2
$r =$	1	0
$p =$	9	8

$10 a^9 c = 210 \rightarrow \boxed{a^9 c = 21}$

$\frac{9\times10}{2} a^8 b^2 = 210 \rightarrow \boxed{a^8 b^2 = \frac{\cancel{210}^{14}}{\cancel{45}_{3}}}$

$a^9 b = 2 \qquad a^9 c = 21 \qquad a^8 b^2 = \frac{14}{3}$

$$\boxed{a = 1 \quad b = 2 \qquad c = 21}$$

$a+b+c = 24$

$b = \frac{2}{a^9}$

$a^8 \cdot \frac{2^2}{a^{18}} = \frac{14}{3}$

$\frac{4}{a^{10}} = \frac{14}{3}$

$a = \left(\frac{3}{7}\right)^{1/10} \qquad b = \frac{2}{\left(\frac{3}{7}\right)^{9/10}}$