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Maxima and Minima of functions

① AM $\geq$ GM.

Sum of numbers $\geq n$ (product of the numbers)$^{1/n}$.

If the sum is given then the product is MAXIMUM when the numbers are equal.

If the product is given then the sum is MINIMUM when the numbers are equal.

Eg $\quad a+b+c = 15 \quad \therefore (abc)_{max} = 5\times5\times5 = 125$

$a=b=c=5$

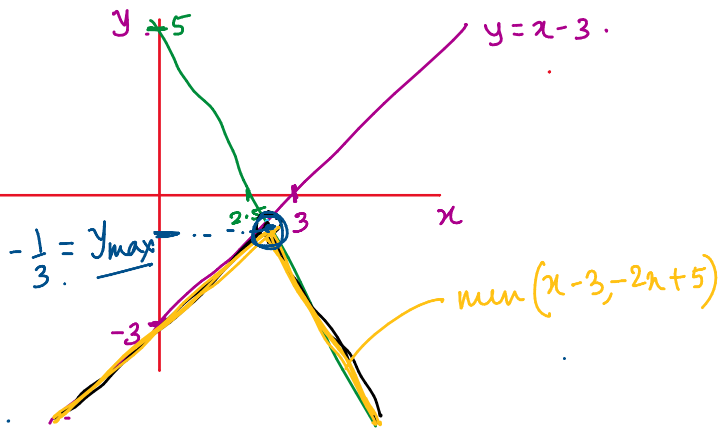
$abc = 216. \quad \therefore (a+b+c)_{min} = 6+6+6 = 18.$

$a=b=c=\sqrt[3]{216} = 6.$

② $y = \min(x-3, -2x+5)$

find $y_{max} = -\frac{1}{3}.$

$y = x-3.$



$y = -2x+5$

$x-3 = -2x+5$

$3x = 8$

$\boxed{x = \frac{8}{3}}$

$y = \frac{8}{3} - 3$

$y = -\frac{1}{3}.$

$y = \min(x^2+x-3, 2x+3) \quad y_{max} = ? = 9$

step1. Equate x^2+x-3 and $2x+3$ to find the points of intersection

$x^2+x-3 = 2x+3.$

$x^2-x-6 = 0$

$(x-3)(x+2) = 0$

$\boxed{x = 3, -2}$

step2 find the value of y by using the value of x.

$x = 3 \quad 2x+3 = 9 \quad x^2+x-3 = 9+3-3 = 9$

step 2 find the value of y by using ...

$x = 3 \quad 2x+3 = 9 \quad x^2+x-3 = 9+3-3 = 9$

$x = -2 \quad 2x+3 = -1 \quad x^2+x-3 = 4-2-3 = -1$

3 Express the given function as the sum or difference of 2 square.

$y = x^2+x+5$. find y_{min}.

$y = \left[x^2 + 2\times\frac{1}{2}\times x + \left(\frac{1}{2}\right)^2\right] - \left(\frac{1}{2}\right)^2 + 5$

$y = \left(x+\frac{1}{2}\right)^2 + \left(5-\frac{1}{4}\right) = \left(x+\frac{1}{2}\right)^2 + \frac{19}{4}$

≥ 0

min value of $(x+\frac{1}{2})^2 = 0$.

$y_{min} = \frac{19}{4}$.

4 If more than one variable is involved then to maximize one of them minimize the others and vice versa.

The average of 5 natural numbers is 20. If all the numbers are distinct then find the maximum value of the lowest possible number and the minimum value of the largest possible number.

$a+b+c+d+e = 100$. avg = 20 ∴ sum = 100.

$1+2+3+4+90$ — 99

$4+5+6+7+76$ — 91.

$20+20+20+20+20$.

hint

14 20 21 22 23

15 19 21 22 23

18 19 20 21 22

To maximize the lowest number and minimize the largest no take the middle no = avg of all the numbers and then keep adding +1 to each number on the right and subtract −1 from each no to the left.

$a+b+c+d+e = 100$. maximize e. (largest)

$1+2+3+4+90 = e_{max}$

$2x + 3y = 100$. and x, y are natural numbers., $x \geq y$.

$2x + 3y = 100$ and x, y are natural numbers, ...

find x_{min}, x_{max}, y_{min}, y_{max}.

$=20$ ... $=20$

$2x + 3y = 100$

$\boxed{2x} + 3y = \boxed{100}$

even. even. even.

$\boxed{y = \text{even}}$

$y_{min} = 2$

$x_{max} = \frac{100 - 3\times 2}{2}$

$x_{max} = 47$

$x \to$ AP with cd $= -3$.

$y \to$ AP with cd $= 2$.

x	y
47	2
44	4
41	6
.	.
.	.
.	.
$50-3n \geq$	$2n$
20	20

-3, -3 ; $+2$, $+2$

t_n

$2x = 100 - 3y$.

$x = 50 - \frac{3y}{2}$

$t_n = a + (n-1)d$

$x = 47 + (n-1)(-3) = 50 - 3n$

$y = 2 + (n-1)2 = 2n$.

$50 - 3n \geq 2n$.

$50 \geq 5n$ $\quad n \leq 10$

5. Using the trigonometric identities to maximize or minimize.

$-1 \leq \sin x \leq 1$
or
$\cos x$.

$-\sqrt{a^2+b^2} \leq a\sin x + b\cos x \leq \sqrt{a^2+b^2}$

$y = a\sin x + b\cos x$.

multiply and divide by $\sqrt{a^2+b^2}$

$y = \sqrt{a^2+b^2}\left[\frac{a\sin x}{\sqrt{a^2+b^2}} + \frac{b\cos x}{\sqrt{a^2+b^2}}\right]$

$y = \sqrt{a^2+b^2}\left[\underbrace{\frac{a}{\sqrt{a^2+b^2}}}_{\cos\alpha}\sin x + \underbrace{\frac{b}{\sqrt{a^2+b^2}}}_{\sin\alpha}\cos x\right] = \sqrt{a^2+b^2}(\sin x\cos\alpha + \cos x\sin\alpha)$

$b^2 > 0$

$a^2 + b^2 > a^2$.

$\frac{a^2}{a^2+b^2} < 1$

$\frac{a}{\sqrt{a^2+b^2}} < 1$

$\therefore \frac{a}{\sqrt{a^2+b^2}}$ is a fraction.

let $\frac{a}{\sqrt{a^2+b^2}} = \cos\alpha$.

$\sin\alpha = \sqrt{1 - \cos^2\alpha}$.

$= \sqrt{1 - \frac{a^2}{a^2+b^2}}$

$= \sqrt{\frac{b^2}{a^2+b^2}} = \frac{b}{\sqrt{a^2+b^2}}$

$y = \sqrt{a^2+b^2}\,\boxed{\sin(x+\alpha)}$ $\quad -1 \leq \sin(x+\alpha) \leq 1$

$-\sqrt{a^2+b^2} \leq \sqrt{a^2+b^2}\sin(x+\alpha) \leq \sqrt{a^2+b^2}$

$\underbrace{-\sqrt{a^2+b^2}}_{\text{min}} \leq a\sin x + b\cos x \leq \underbrace{\sqrt{a^2+b^2}}_{\text{max}}$

$z = 5x + 6y$. where x and y are sin and cos of some angle

find the max and min value of z.

$$-\sqrt{5^2+6^2} \leq z = 5\sin\theta + 6\cos\theta \leq \sqrt{5^2+6^2}$$