09 March 2024 09:08 PM

Integration → Application in Economics.

① $TC = C(Q)$

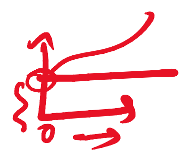
$AC = \frac{TC}{Q}$

$$\boxed{MC = \frac{dTC}{dQ}}$$

$$\int MC \cdot dQ = TC$$

From TC to MC ⇒ differentiate

MC to TC ⇒ integrate.



$TC = TVC + TFC$

$Q = 0$ $TVC = 0$

$\therefore TC = TFC$

Ex : $MC = 3Q$. find TC. when product is 0, then TC is 20.

We know $MC = \frac{dTC}{dQ}$

or, $\frac{dTC}{dQ} = 3Q$

$dTC = 3Q \cdot dQ$

Integrating both sides

$\int dTC = 3\int Q \cdot dQ$

$TC = 3 \cdot \frac{Q^2}{2} + IC$

or, $TC = \frac{3Q^2}{2} + FC$

$\int dx = x$.

IC as FC

IC = FC the fixed cost

$20 = \frac{3 \times 0^2}{2} + fc$

$20 = 0 + fc$

① Total cost from marginal cost

$$\frac{}{2}$$

$$20 = 0 + FC$$

$$FC = 20$$

∴ The required Cost fn is $TC = \frac{3Q^2}{2} + 20$ (ans)

② Total Revenue from Marginal Revenue.

$$TR = P \times Q$$

$$AR = \frac{TR}{Q} = \frac{P \times Q}{Q} = P$$

$\int dx$

$$MR = \frac{dTR}{dQ}$$

∴

$$TR = \int (MR)\, dQ$$

Suppose,

Ex: $MR = 2 - 0.3Q$.

Find the TR function.

$$MR = 2 - 0.3Q$$

or $\frac{dTR}{dQ} = 2 - 0.3Q$

or $\boxed{TR = \int (2 - 0.3Q)\, dQ}$

$$TR = 2Q - \frac{0.3Q^2}{2} + IC$$

we know if $Q = 0$

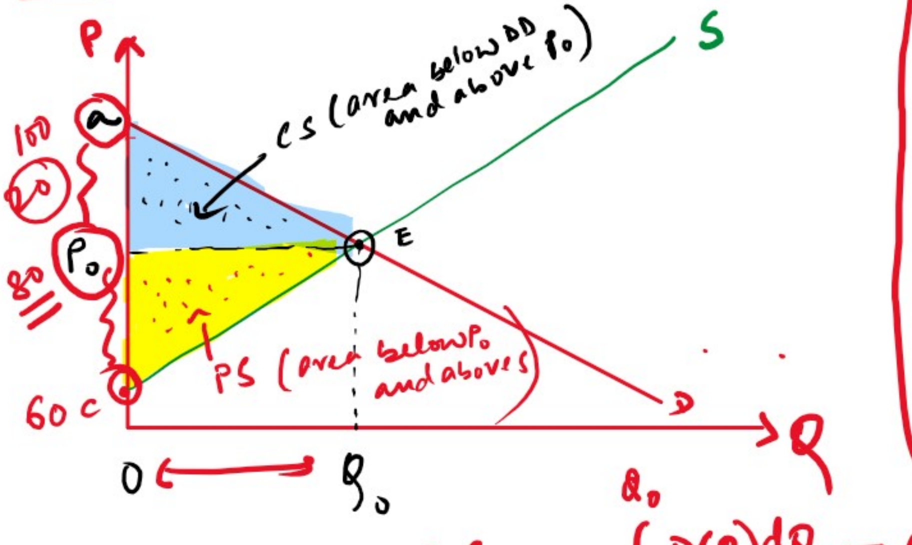
then $TR = 0$

∴ $0 = 0 + IC$

$IC = 0$

∴ $\boxed{TR = 2Q - \frac{0.3Q^2}{2}}$

is the required TR function.

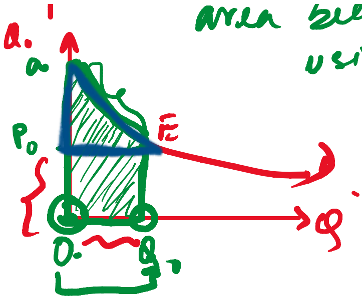


$$CS = \text{area } \triangle aP_0E = \int_0^{Q_0} D(Q)\, dQ - \text{area } OP_0EQ_0$$

area below curve using integration

$CS =$ area $\triangle\, aP_0E = \int_0 D(Q)\,dQ -$ area OP_0EQ_0

area below curve using integrals



$$CS = \int_0^{Q_0} D(Q)\,dQ - (OP_0 \times OQ_0)$$

Ex: Suppose

$P = 32 - 8Q$ is the demand fn

$P = 12 + 2Q$ is the supply fn.

Calculate Consumer Surplus.

In market equilibrium

$D = S$

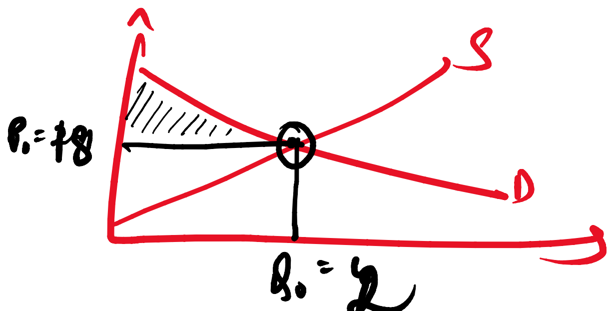
$32 - 8Q = 12 + 2Q$

$32 - 12 = 8Q + 2Q$

$20 = 10Q$

$\boxed{Q = 2 \text{ units}}$

$\therefore P = 12 + 2Q = 12 + 2\times 2$
$= 16$



$$CS = \int_0^{Q_0} D(Q)\,dQ - (OP \times OQ)$$

$$= \int_0^2 (32 - 8Q)\,dQ - (16 \times 2)$$

$$= \int_0^2 32\,dQ - 8\int_0^4 Q\,dQ - 32$$

$$= 32[Q]_0^2 - \cancel{8}\,4\left[\frac{Q^2}{2}\right]_0^2 - 32$$

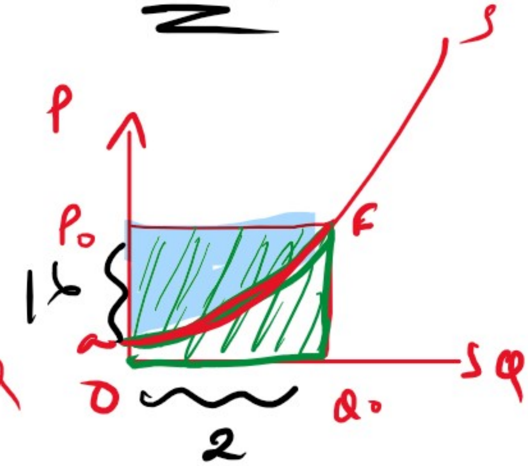
$$= 32(2 - 0) - 4(2^2 - 0^2) - 32$$

$= \left(32 \times 2 - 16\right) - 32$

$= 64 - 16 - 32$

$= 32 - 16 = 16$ (ans)

Producer's surplus :



PS = area $\square OP_0EQ_0 - \int_0^{Q_0} S(Q)\,dQ$

$Q_0 = 2 \qquad P_0 = 16 \qquad OP_0 \times OQ_0 = 32$

$PS = OP_0 \times OQ_0 - \int_0^2 (12 + 2Q)\,dQ$

$= 32 - \int_0^2 12\,dQ - \int_0^2 2Q\,dQ$

$= 32 - 12[Q]_0^2 - \cancel{2}\left[\frac{Q^2}{\cancel{2}}\right]_0^2$

$= 32 - 12(2-0) - [2^2 - 0]$

$= 32 - 24 - 4$

$= 32 - 28$

$= 4$ (ans)