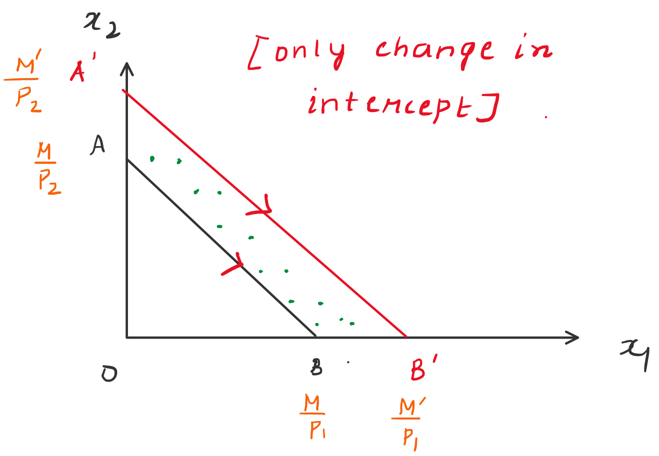
Decomposition of Price Effect (Slutsky Equation):

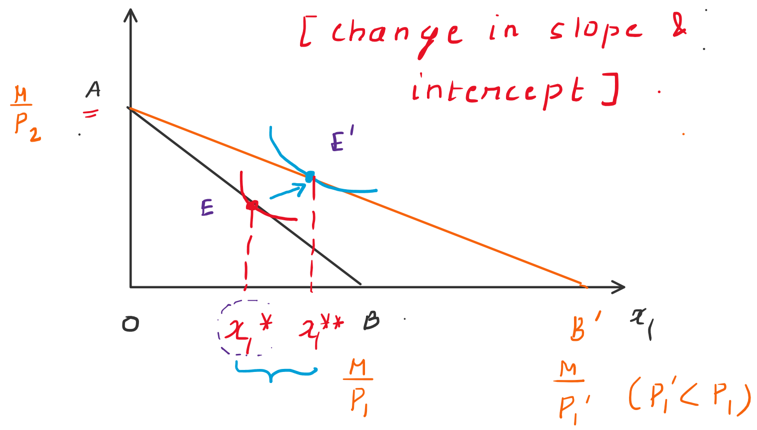
Idea: Budget line: $M = P_1 x_1 + P_2 x_2$. $\left[x_1^{max} = \frac{M}{P_1},\ x_2^{max} = \frac{M}{P_2},\ \left|\frac{dy}{dx}\right| = \frac{P_1}{P_2} \right]$

Case I: $M \uparrow$



Outward shift of the BL
⇒ Increase in purchasing power of consumer.

Case II: $P_1 \downarrow$



$P_1 \downarrow$ ⇒ Good 1 cheaper
Indirectly also increase the purchasing power of the consumer.

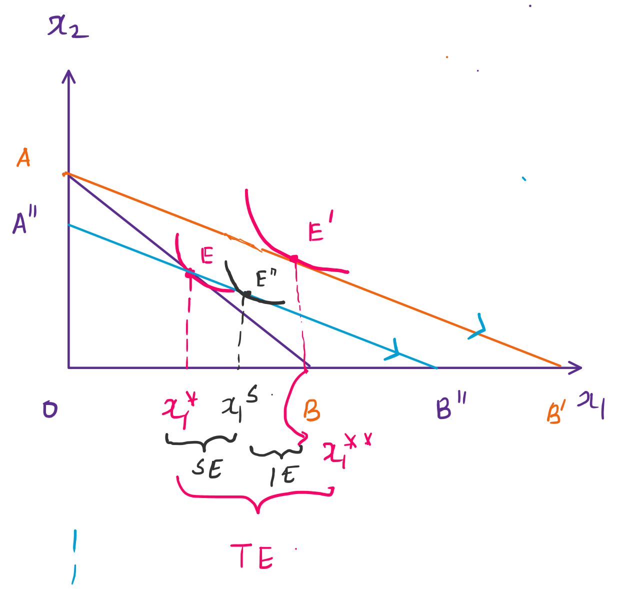
Note: From Fig II as change in P_1 also shifts the BL (which was observed in Fig I), hence change in P_1 has an inbuild income effect as well (which leads to indirect increase of purchasing power).

∴ From the Fig II: $x_1^* \rightarrow x_1^{**}$ (Total price effect) / TE / PE.

⇒ Price Effect = $\underbrace{\text{Substitution Effect}}_{\text{change in demand solely due to change in prices keeping purchasing power constant.}}$ + $\underbrace{\text{Income Effect}}_{\text{Change in demand due to the change in purchasing power.}}$

Price Effect on Good 1: $\boxed{PE = SE + IE} \Rightarrow$ Slutsky Equation.

Note: AB = Old BL.
AB' = New BL.



For SE: Evaluate Δx_1 due to ΔP_1 keeping purchasing power constant.

Push the new BL ||ly backwards till it passes through the old consumption bundle ⇒ consumer has just enough money to afford the old bundle. (given by A'' B'')

[Slutsky SE].

original demand: x_1^*
Final demand: x_1^{**}
$x_1^* \to x_1^S$: SE
$x_1^S \to x_1^{**}$: IE
TE = SE + IE : $x_1^* \to x_1^{**}$

Q. Consumer: $u(x_1, x_2) = x_1^2 x_2^3$ (Cobb Douglas). $M = 1000$, $P_2 = 20$. Suppose P_1 decreases from 5 to 4. Decompose the total effect of change in Good 1.

$$x_1^* = \frac{2}{5} \cdot \frac{M}{P_1} \qquad x_2^* = \frac{3}{5} \cdot \frac{M}{P_2}.$$

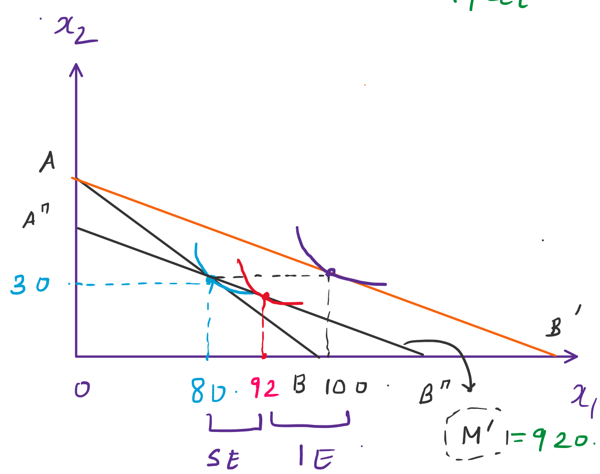
Old Prices: (5, 20):

$$x_1^* = \frac{2}{5} \cdot \frac{1000}{5} = 80.$$

$$x_2^* = \frac{3}{5} \cdot \frac{1000}{5} = 30.$$

Final Prices: (4, 20).

$$x_1^* = \frac{2}{5} \cdot \frac{1000}{4} = 100, \quad x_2^* = 30$$



SE = 92 − 80 = 12.
IE = 100 − 92 = 8.
TE = SE + IE = 20.

New BL: A''B''. Find M': money income leaves the old bundle

$\overline{5} \cdot \overline{4}$

New BL $A''B''$ Find M': money income leaves the old bundle exactly affordable at new prices.

M' = old bundle exactly affordable at new prices.

Old Bundle: $(x_1^*, x_2^*) = (80, 30)$

New Prices: $(P_1', P_2) = (4, 20)$

$$M' = 80 \times 4 + 30 \times 20 = 320 + 600 = 920$$

$$x_1^S = \frac{2}{5} \cdot \frac{M'}{P_1'} = \frac{2}{5} \cdot \frac{920}{4} = \frac{2 \times 92\cancel{0}}{2\cancel{0}} = 92$$

HW.

Q. $U(x_1, x_2) = x_1^2 x_2^3$. $M = 1000$, $P_2 = 20$. Suppose Price of Good 1 increases from 4 to 5. Decompose the total effect on Good 1. Show it graphically as well.