ECONOMICS SESSION 24.08.2023

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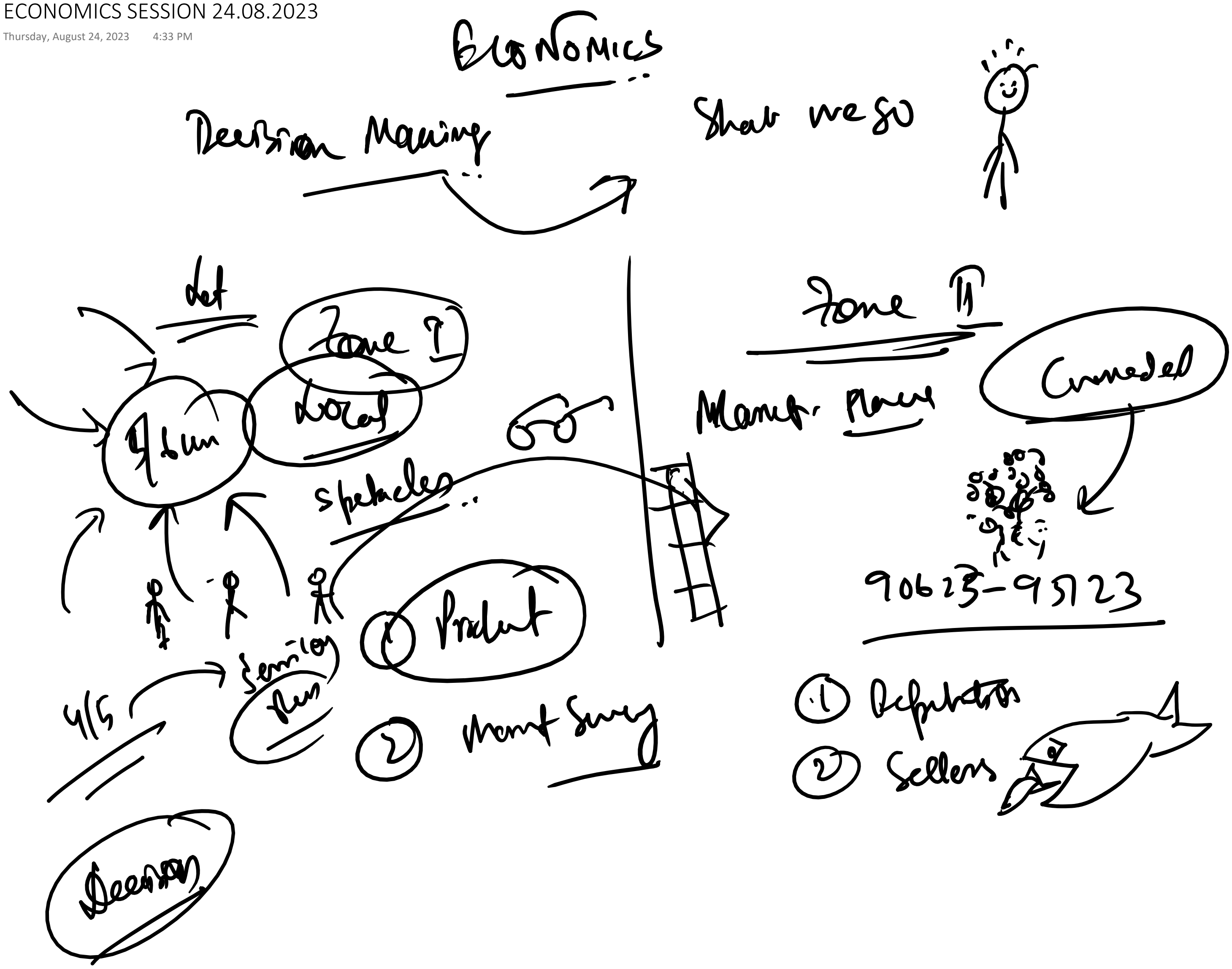


ECONOMIC
S SESSION...

2. Show that two utility functions given below generate the identical demand functions for goods X and Y:

a. $U(X, Y) = \log(X) + \log(Y)$

b. $U(X, Y) = (XY)^{0.5}$

$p_x x + p_y y = I$

$L = \ln x + \ln y - \lambda(p_x x + p_y y - I)$

$\frac{\partial L}{\partial x} = \frac{1}{x} - \lambda p_x = 0$

$\frac{\partial L}{\partial y} = \frac{1}{y} - \lambda p_y = 0$

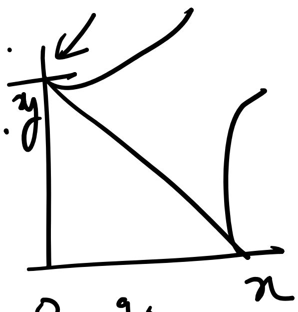
$\frac{\partial L}{\partial \lambda} = p_x x + p_y y - I = 0$

$p_x x = p_y y$

$X = \frac{0.5}{p_x} I \qquad Y = \frac{0.5}{p_y} I$

Economic Interpretation: the consumer values both products equally the same

Cross price elasticity



4. Sharon has the following utility function:

$$U(X,Y) = \sqrt{X} + \sqrt{Y}$$

where X is her consumption of candy bars, with price P_X=$1, and Y is her consumption of espressos, with P_Y=$3.

a. Derive Sharon's demand for candy bars and espressos.

b $L = \sqrt{x} + \sqrt{y} - \lambda(P_x X + P_y Y - I)$

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X,Y equating the $f_x\ y$ ..

$\frac{\partial L}{\partial x} = \frac{1}{2\sqrt{x}} - P_x \lambda = 0$ $\quad \frac{1}{2\sqrt{x} P_x} = \lambda$

$\frac{\partial L}{\partial y} = \frac{1}{2\sqrt{y}} - P_y \lambda = 0$ $\quad \frac{1}{2\sqrt{y} P_y} = \lambda$

$\frac{\partial L}{\partial \lambda} = P_x X + P_y Y - I = 0$

$X = \frac{P_x \cdot I}{}$

$\frac{1}{2\sqrt{x} \cdot P_x} = \frac{1}{2\sqrt{y} P_y}$

$X = \left(\frac{P_y^2}{P_x^2}\right) Y$

$X P_x^2 = Y P_y^2$

$X P_x \cdot P_x = Y \cdot P_y \cdot P_y$

$Y = \frac{P_x I}{P_y^2 + P_y P_x}$ or

$Y = \frac{I}{12}$ $\quad X = \frac{3I}{4}$

b. Assume that her income I=$100. How many candy bars and espressos will Sharon consume?

$Y = \frac{100}{12} = 8.3$ $\quad X = \frac{3 \cdot 100}{4} = 75$

c. What is the marginal utility of income?

$\lambda = \frac{1}{2P_x\sqrt{x}} = \frac{1}{2P_y\sqrt{y}}$

Substituting each other price into the eqn

$\lambda = 0.058$

Here λ = MU of income

5. **Maurice has the following utility function: $U(X,Y) = 20X + 80Y - X^2 - 2Y^2$, where X is his consumption of CD's, with a price of \$1, and Y is his consumption of movie videos, with a rental price of \$2. He plans to spend \$41 on both forms of entertainment. Determine the number of CD's and video rentals that will maximize Maurice's utility.**

$I = 41 \quad P_x = 1 \quad P_y = 2$

Same as before..

$L = 20x + 80y - x^2 - 2y^2 - \lambda(x + 2y - 41)$

x, y finally..

Expected Uncertainty Consumer Behavior..

4. Suppose an investor is concerned about a business choice in which there are three prospects, whose probability and returns are given below:

Probability	Return
0.4	$100
0.3	30
0.3	-30

What is the expected value of the uncertain investment? What is the variance?

AMC var-calc is imp..

EV = prob × Return

$= (0.4 \times 100) + (0.3 \times 30) + (0.3 \times (-30))$

$= 40 + 3 - 3 = 40$

$\sigma^2 = 0.4(100-40)^2 + 0.3(30-40)^2 + 0.3(-30-40)^2$

→

5. You are an insurance agent who has to write a policy for a new client named Sam. His company, Society for Creative Alternatives to Mayonnaise (SCAM), is working on a low-fat, low-cholesterol mayonnaise substitute for the sandwich condiment industry. The sandwich industry will pay top dollar to whoever invents such a mayonnaise substitute first. Sam's SCAM seems like a very risky proposition to you. You have calculated his possible returns table as follows.

Probability	Return	
.999	-\$1,000,000	(he fails) 10 Lakh
.001	\$1,000,000,000	(he succeeds and sells the formula) 100 Crore

a. What is the expected return of his project? What is the variance?

$ER = 0.999(-1000000) + 0.001(10^9)$

$= 1000 > 0$

$\sigma^2 = 0.999(-10^6 - 10^3)^2 + 0.001(10^9 - 10^3)^2$

$\sigma^2 =$

Expected Return

0.5 0.5

300 0 100

$300 \times 0.5 + 0.5 \times 0$

$= 150 + 0$

$= 150$

ER > Inv

b. What is the most Sam is willing to pay for insurance? Assume Sam is risk neutral.

Expected Income ₹1000

Expected Income $1000

As E.I < 0 so chances that he will end up ≥ 0 income so he would be reluctant to buy the insurance..

Health Insurance

USA / UK

$10000

c. Suppose you found out that the Japanese are on the verge of introducing their own mayonnaise substitute next month. Sam does not know this and has just turned down your final offer of $1,000 for the insurance. Assume that Sam tells you SCAM is only six months away from perfecting its mayonnaise substitute *and* that you know what you know about the Japanese. Would you raise or lower your policy premium on any subsequent proposal to Sam? Based on his information, would Sam accept?

Japanese → M → Lowers Sam's prob of a high payoff.

Billion dollar payoff → 0

expected income ↓ Sales ↓

Premium ↑↑

But, Sam will Reject

I ↑ Sam

6. Suppose that Natasha's utility function is given by $u(I) = \sqrt{10I}$, where I represents annual income in thousands of dollars.

a. Is Natasha risk loving, risk neutral, or risk averse? Explain.

Risk Averse

$$\frac{dU}{dI} = \frac{1}{\sqrt{10I}} = \frac{1}{\sqrt{10}} I^{-\frac{1}{2}} > 0$$

$$\frac{d^2U}{dI^2} = -\left(\frac{1}{\sqrt{10}} \cdot \frac{1}{2} I^{-3/2}\right) < 0$$

b. Suppose that Natasha is currently earning an income of $40,000 ($I = 40$) and can earn that income next year with certainty. She is offered a chance to take a new job that offers a .6 probability of earning $44,000, and a .4 probability of earning $33,000. Should she take the new job?

$40^{0.5} \to 20$

$EU = 0.6(44)^{1/2} + 0.4(33)^{1/2}$

$= 19.85$

$I \to \sqrt{10I} = 20$

Won't take the job

c. In (b), would Natasha be willing to buy insurance to protect against the variable income associated with the new job? If so, how much would she be willing to pay for that insurance? (Hint: What is the risk premium?)

New Job →

Risk Premium → Diff b/w the two Payments

40000
39410
00590

$19.85 = (10I)^{0.5}$

$I = 39410$

60000
20000
→ 2,00,000
10%

Insurance → 1 Crore → 70 Lakh

30 Lakh / 10% → 3 Lakh

7. Suppose that two investments have the same three payoffs, but the probability associated with each payoff differs, as illustrated in the table below:

Payoff	Probabilities for Investment A	Probabilities for Investment B
$300	0.10	0.30
$250	0.80	0.40
$200	0.10	0.30

a. Find the expected return and standard deviation of each investment.

E.V. = 250

a. Find the expected return and standard deviation of each investment.

EV_A = 250 EVB = 250

$EU_A = 0.1(5\times300) + 0.8(5\times250) + 0.1(5\times200)$

$= 1250$

$EU_B = 0.3(5\times300) + 0.4(5\times250) + 0.3(5\times200)$

$= 1250$

~~The~~ ~~Gee~~ Jill will be indifferent between the two of the cases.

b. Jill has the utility function $U = 5I$, where I denotes the payoff. Which investment will she choose?

Indifferent

c. *Ken has the utility function* $U = \sqrt{5I}$. **Which investment will he choose?**

d. **Laura has the utility function $U = 5I^2$. Which investment will she choose?**

$EU = 0.1(5 \times 3w^2) + 0.8(5 \times 30^2) + 0.1(5 \times 2n^2)$

→ Calc..

8. As the owner of a family farm whose wealth is $250,000, you must choose between sitting this season out and investing last year's earnings ($200,000) in a safe money market fund paying 5.0% or planting summer corn. Planting costs $200,000, with a six-month time to harvest. If there is rain, planting summer corn will yield $500,000 in revenues at harvest. If there is a drought, planting will yield $50,000 in revenues at harvest. As a third choice, you can purchase AgriCorp drought-resistant summer corn at a cost of $250,000 that will yield $500,000 in revenues at harvest if there is rain, and $350,000 in revenues at harvest if there is a drought. You are risk averse and your preferences for family wealth (W) are specified by the relationship $U(W) = \sqrt{W}$. The probability of a summer drought is 0.30 and the probability of summer rain is 0.70. Which of the three options should you choose? Explain.

8. You manage a plant that mass produces engines by teams of workers using assembly machines. The technology is summarized by the production function.

$$q = 5\,KL$$

where q is the number of engines per week, K is the number of assembly machines, and L is the number of labor teams. Each assembly machine rents for r = \$10,000 per week and each team costs w = \$5,000 per week. Engine costs are given by the cost of labor teams and machines, plus \$2,000 per engine for raw materials. Your plant has a fixed installation of 5 assembly machines as part of its design.

a. What is the cost function for your plant — namely, how much would it cost to produce q engines? What are average and marginal costs for producing q engines? How do average costs vary with output?

b. How many teams are required to produce 250 engines? What is the average cost per engine?

c. You are asked to make recommendations for the design of a new production facility. What capital/labor (K/L) ratio should the new plant accommodate if it wants to minimize the total cost of producing any level of output q?

4. Suppose the process of producing light-weight parkas by Polly's Parkas is described by the function:

$$q = 10K^{.8}(L - 40)^{.2}$$

where q is the number of parkas produced, K the number of computerized stitching-machine hours, and L the number of person-hours of labor. In addition to capital and labor, \$10 worth of raw materials are used in the production of each parka.

a. By minimizing cost subject to the production function, derive the cost-minimizing demands for K and L as a function of output (q), wage rates (w), and rental rates on machines (r). Use these results to derive the total cost function, that is costs as a function of q, r, w, and the constant \$10 per unit materials cost.

b. This process requires skilled workers, who earn $32 per hour. The rental rate on the machines used in the process is $64 per hour. At these factor prices, what are total costs as a function of q? Does this technology exhibit decreasing, constant, or increasing returns to scale?

c. Polly's Parkas plans to produce 2000 parkas per week. At the factor prices given above, how many workers should the firm hire (at 40 hours per week) and how many machines should it rent (at 40 machines-hours per week)? What are the marginal and average costs at this level of production?

10. Suppose you are given the following information about a particular industry:

$Q^D = 6500 - 100P$	Market demand
$Q^S = 1200P$	Market supply
$C(q) = 722 + \frac{q^2}{200}$	Firm total cost function
$MC(q) = \frac{2q}{200}$	Firm marginal cost function.

Assume that all firms are identical, and that the market is characterized by pure competition.

a. Find the equilibrium price, the equilibrium quantity, the output supplied by the firm, and the profit of the firm.

b. Would you expect to see entry into or exit from the industry in the long-run? Explain. What effect will entry or exit have on market equilibrium?

c. What is the lowest price at which each firm would sell its output in the long run? Is profit positive, negative, or zero at this price? Explain.

d. What is the lowest price at which each firm would sell its output in the short run? Is profit positive, negative, or zero at this price? Explain.

4. A firm faces the following average revenue (demand) curve:

$$P = 120 - 0.02Q$$

where Q is weekly production and P is price, measured in cents per unit. The firm's cost function is given by $C = 60Q + 25{,}000$. Assume that the firm maximizes profits.

a. What is the level of production, price, and total profit per week?

b. If the government decides to levy a tax of 14 cents per unit on this product, what will be the new level of production, price, and profit?

8. A firm has two factories for which costs are given by:

$$\text{Factory \#1: } C_1(Q_1) = 10Q_1^2$$

$$\text{Factory \#2: } C_2(Q_2) = 20Q_2^2$$

The firm faces the following demand curve:

$$P = 700 - 5Q$$

where Q is total output, i.e. $Q = Q_1 + Q_2$.

a. On a diagram, draw the marginal cost curves for the two factories, the average and marginal revenue curves, and the total marginal cost curve (i.e., the marginal cost of producing $Q = Q_1 + Q_2$). Indicate the profit-maximizing output for each factory, total output, and price.

b. Calculate the values of Q_1, Q_2, Q, and P that maximize profit.

c. Suppose labor costs increase in Factory 1 but not in Factory 2. How should the firm adjust the following(i.e., raise, lower, or leave unchanged): Output in Factory 1? Output in Factory 2? Total output? Price?

12. Michelle's Monopoly Mutant Turtles (MMMT) has the exclusive right to sell Mutant Turtle t-shirts in the United States. The demand for these t-shirts is $Q = 10{,}000/P^2$. The firm's short-run cost is SRTC = 2,000 + 5Q, and its long-run cost is LRTC = 6Q.

a. What price should MMMT charge to maximize profit in the short run? What quantity does it sell, and how much profit does it make? Would it be better off shutting down in the short run?

b. What price should MMMT charge in the long run? What quantity does it sell and how much profit does it make? Would it be better off shutting down in the long run?

c. Can we expect MMMT to have lower marginal cost in the short run than in the long run? Explain why.

15. Dayna's Doorstops, Inc. (DD), is a monopolist in the doorstop industry. Its cost is $C = 100 - 5Q + Q^2$, and demand is $P = 55 - 2Q$.

a. What price should DD set to maximize profit? What output does the firm produce? How much profit and consumer surplus does DD generate?

b. What would output be if DD acted like a perfect competitor and set MC = P?
What profit and consumer surplus would then be generated?

c. ***What is the deadweight loss from monopoly power in part (a)?***

d. Suppose the government, concerned about the high price of doorstops, sets a maximum price at $27. How does this affect price, quantity, consumer surplus, and DD's profit? What is the resulting deadweight loss?

e. Now suppose the government sets the maximum price at $23. How does this affect price, quantity, consumer surplus, DD's profit, and deadweight loss?

f. **Finally, consider a maximum price of $12. What will this do to quantity, consumer surplus, profit, and deadweight loss?**

*16. There are 10 households in Lake Wobegon, Minnesota, each with a demand for electricity of Q = 50 - P. Lake Wobegon Electric's (LWE) cost of producing electricity is TC = 500 + Q.

a. If the regulators of LWE want to make sure that there is no deadweight loss in this market, what price will they force LWE to charge? What will output be in that case? Calculate consumer surplus and LWE's profit with that price.

b. If regulators want to ensure that LWE doesn't lose money, what is the lowest price they can impose? Calculate output, consumer surplus, and profit. Is there any deadweight loss?

c. Kristina knows that deadweight loss is something that this small town can do without. She suggests that each household be required to pay a fixed amount just to receive any electricity at all, and then a per-unit charge for electricity. Then LWE can break even while charging the price you calculated in part (a). What fixed amount would each household have to pay for Kristina's plan to work? Why can you be sure that no household will choose instead to refuse the payment and go without electricity?

18. A monopolist faces the following demand curve:

$$Q = 144/P^2$$

where Q is the quantity demanded and P is price. Its *average variable* cost is

$$AVC = Q^{1/2},$$

and its *fixed cost* is 5.

a. What are its profit-maximizing price and quantity? What is the resulting profit?

b. Suppose the government regulates the price to be no greater than $4 per unit. How much will the monopolist produce? What will its profit be?

c. Suppose the government wants to set a ceiling price that induces the monopolist to produce the largest possible output. What price will accomplish this goal?