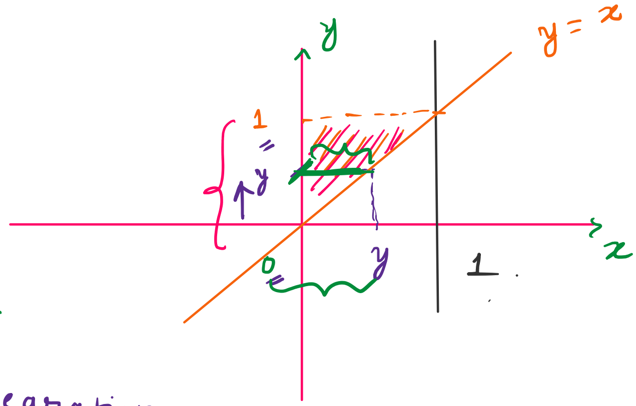
Q. $\int_0^1 \int_x^1 y^4 e^{xy^2}\, dy\, dx$. [Int w.r.t y & then w.r.t x]

$x \in [0,1]$, $y \in [x,1]$.

$= \int_0^1 dx \int_x^1 y^4 e^{xy^2} dy$.



We would "prefer" to perform integration w.r.t 'x' first.

$$\int_0^1 \int_x^1 y^4 e^{xy^2}\, dy\, dx = \int_0^1 \int_0^y y^4 e^{xy^2}\, dx\, dy, \quad y \in [0,1].$$

$$= \int_0^1 dy \int_0^y y^4 e^{xy^2} dx$$

$$= \int_0^1 dy \cdot y^4 \left[\frac{e^{xy^2}}{y^2}\right]_0^y$$

$$= \int_0^1 dy \cdot y^2 \left[e^{xy^2}\right]_0^y$$

$$= \int_0^1 dy \cdot y^2 \left[e^{y^3} - 1\right]$$

$$= \int_0^1 y^2 \left[e^{y^3} - 1\right] dy.$$

Let $y^3 = z$, $y = 0,\ z = 0$

$y^2 dy = \frac{dz}{3}$, $y = 1,\ z = 1$

$$= \frac{1}{3} \int (e^z - 1)\, dz.$$

$$= \frac{1}{3}\int_0 (e^z - 1)\, dz$$

$$= \frac{1}{3}\left[e^z - z\right]_0^1 = \frac{1}{3}\left[(e-1) - (1-0)\right]$$

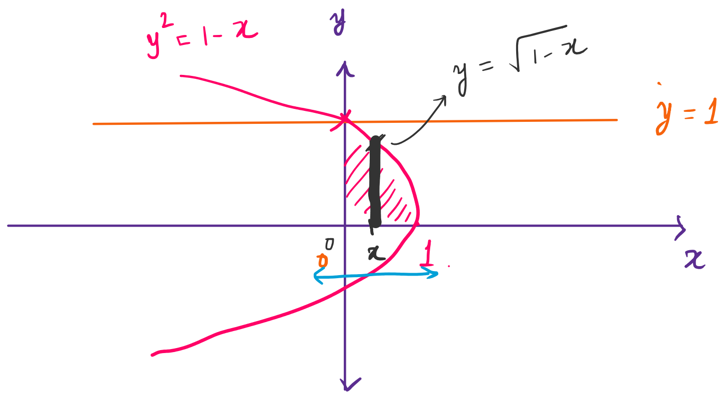
$$= \left(\frac{e-2}{3}\right)$$

8. $\displaystyle\int_0^1\int_0^{1-y^2} y\sin\{\pi(1-x)^2\}\,dx\,dy$, $x \in [0, 1-y^2]$, $y \in [0,1]$

~~(a)~~ $\frac{1}{2\pi}$ (b) 2π (c) $\frac{\pi}{2}$ (d) $\frac{2}{\pi}$

$x = 1 - y^2$

$y^2 = -(x-1)$



$$\int_0^1\int_0^{\sqrt{1-x}} y\sin\{\pi(1-x)^2\}\,dy\cdot dx$$

$$= \int_0^1 dx \cdot \sin\{\pi(1-x)^2\}\cdot\int_0^{\sqrt{1-x}} y\cdot dy$$

$$= \int_0^1 dx\cdot \sin\{\pi(1-x)^2\}\cdot\left[\frac{y^2}{2}\right]_0^{\sqrt{1-x}}$$

$$= \frac{1}{2}\int_0^1 dx\cdot \sin\{\pi(1-x)^2\}(1-x)$$

$$= \frac{1}{2}\int_0^1 (1-x)\cdot\sin\{\pi(1-x)^2\}\cdot dx$$

Let $\pi(1-x)^2 = z$; $x = 0$, $z = \pi$

$$2\pi(1-x)(-1)\cdot dx = dz \qquad\qquad x=1,\; z=0$$

$$(1-x)\,dx = -\frac{1}{2\pi}\,dz$$

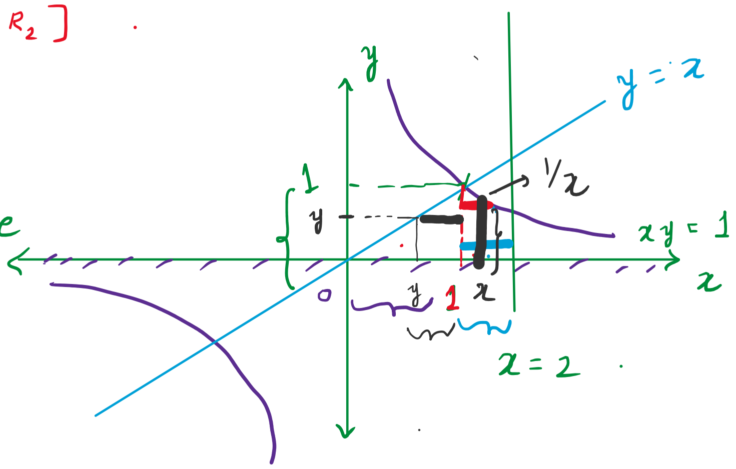
$$= \frac{1}{2}\left(-\frac{1}{2\pi}\right)\int_{\pi}^{0} \sin z\cdot dz = \frac{1}{4\pi}\int_{0}^{\pi} \sin z\,dz$$

$$= \frac{1}{4\pi}\left(-\cos z\right)_{0}^{\pi} = -\frac{1}{4\pi}\left[\cos\pi - \cos 0\right].$$

$$= -\frac{1}{4\pi}\left[-1-1\right] = \frac{2}{4\pi} = \frac{1}{2\pi} \quad (a)$$

$[R = R_1 + R_2]$.

Q. $\iint_R f(x,y)\,dx\,dy$ where R is the region bounded by the curves: $xy=1$, $y=0$, $y=x$, $x=2$.



$$\iint_R f(x,y)\,dx\,dy = \underbrace{\iint_{R_1} \cdots dx\,dy}_{} + \iint_{R_2} \cdots\; dy\,dx.$$

$y \in [0,1]$

$x \in [y,1]$.

$x \in [1,2]$

$y \in [0, 1/x]$.

$$= \int_0^1\int_y^1 f(x,y)\,dx\,dy + \int_1^2\int_0^{1/x} f(x,y)\cdot dy\,dx.$$

HW $f(x,y) = x^2 + y^2$ & solve the integral.