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Measures of Central Tendency

Average

1. AM $(\bar{x})$
2. G.M $(\bar{x}_g)$
3. H.M $(\bar{x}_h)$

Median

Mode

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(1) Arithmetic Mean $(\bar{x})$

(a) Simple AM $(\bar{x})$

Let $x_1, x_2 \ldots x_n$ be n number of observations then

$$\text{Simple mean}(\bar{x}) = \frac{\sum_{i=1}^{n} x_i}{n}$$

(b) Weighted mean $(\bar{x})$

Let $x_1, x_2 \ldots x_n$ be n number of observations with $f_1, f_2 - f_n$ their corresponding frequencies then, of obs...

corresponding frequencies then,

Weighted AM $(\bar{x}) = \dfrac{1}{\sum f_i} \sum_{i=1}^{n} x_i f_i$

where $\sum f_i = N =$ total frequency

Eg: x: 8, 1, 6 $\Rightarrow n = 3$

$\Rightarrow \sum_{i=1}^{3} x_i = 8+1+6 = 15$

f: 3, 2, 5

$\therefore \bar{x} = \frac{1}{n} \sum x_i = \frac{15}{3} = 5$ (ans)

x	f	xf
8	3	24
1	2	2
6	5	30
	$N = \sum f = 10$	$\sum xf = 56$

$\therefore$ Weighted mean

$\bar{x} = \frac{1}{N} \sum xf$

$= \frac{1}{10} \times 56$

$= 5.6$ (ans)

Properties

(1) If all observations are same, then mean is the same value.

Proof: Let $x = c$ for all value of x

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$i = 1(1)n$

then simple mean, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$= \frac{1}{n}\sum_{i=1}^{n} c$

$\bar{x} = \frac{1}{\cancel{n}} \times \cancel{n}c = c$ (proved)

② If all observations are repeated same no. of times ⇒ simple AM = weighted AM.

Let $x_1, x_2 \dots x_n$ be n number of observation with corresponding frequencies $f_1, f_2 \dots f_n$ such that

$f_1 = f_2 = \dots = f_n = k$ (const)

$\therefore N = \sum_{i=1}^{n} f_i = \sum_{i=1}^{n} k = nk$ —— (1)

$\therefore$ Weighted AM $\bar{x} = \frac{1}{N}\sum_{i=1}^{n} x_i f_i$

$= \frac{1}{nk}\sum_{i=1}^{n} x_i k$

$= \frac{\cancel{k}}{n\cancel{k}}\sum_{i=1}^{n} x_i = \frac{1}{n}\sum_{i=1}^{n} x_i$

= simple mean.

$n\bar{x}$

$=$ simple mean

(3) Sum of deviation of obs from it's mean is 0.

$$\sum_{i=1}^{n}(x_i-\bar{x}) = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n}\bar{x}$$

we know $\bar{x} = \frac{1}{n}\sum x_i$

$\sum x_i = n\bar{x}$ (1)

from (1)

$$\sum(x_i-\bar{x}) = n\bar{x} - n\bar{x}$$

$$= 0$$ proved

(4) AM is dependent on change in both origin and scale

In other words, if all observations are changed by same amount or same proportion or both, then it's AM will change by same amount and same proportion.

Proof: all obs x is increased by 'a' (the origin) and ~~chang~~ multiplied by 'b' (the scale) then it can be ...

by 'b' (the scale) then it can be written as $y_i = a + bx_i$

Now $\bar{y} = \frac{1}{n}\sum_{i=1}^{n}(y_i)$

or, $\bar{y} = \frac{1}{n}\sum_{i=1}^{n}(a+bx_i)$

or, $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} a + \frac{b}{n}\sum_{i=1}^{n} x_i$

or, $\bar{y} = \frac{\cancel{n}a}{\cancel{n}} + b\bar{x}$

or, $\boxed{\bar{y} = a + b\bar{x}}$ Proved

Prop 5 sum of square of deviation of obs from its mean is least.

ie $\sum_{i=1}^{n}(x_i - A)^2$ is min when $A = \bar{x}$.

Proof : Let $\sum_{i=1}^{n}(x_i - A)^2 = \sum_{i=1}^{n}\left[(x_i - \bar{x}) + (\bar{x} - A)\right]^2$

$= \sum_{i=1}^{n}\left\{(x_i - \bar{x})^2 + 2(x_i - \bar{x})(\bar{x} - A) \quad \ldots)^2\right\}$

$$= \sum_{i=1} \left\{ (x_i - \bar{x}) + 2(x_i - \bar{x})(\bar{x} - A) + (\bar{x} - A)^2 \right\}$$

$$= \sum (x_i - \bar{x})^2 + 2(\bar{x} - A)\sum(x_i - \bar{x}) + \sum_{i=1}^{n} (\bar{x} - A)^2$$

$$\sum_{i=1}^{n} (x_i - A)^2 = \sum (x_i - \bar{x})^2 + n(\bar{x} - A)^2$$

zero or positive.

$$n(\bar{x} - A)^2 = 0 \quad \text{(min value)}$$

$$\bar{x} - A = 0$$

$$\bar{x} = A$$ the $\sum (x_i - A)^2$ is minimised

Geometric Mean

a) simple GM $(\bar{x}_g) = (x_1 \cdot x_2 \cdots x_n)^{1/n}$

$$= \left(\prod_{i=1}^{n} x_i \right)^{1/n}$$

(b) weighted GM $(\bar{x}_g) = \left(x_1^{f_1} \cdot x_2^{f_2} \cdot x_3^{f_3} \cdots x_n^{f_n} \right)^{1/N}$

$\left(\prod^{n} f_i \right)^{1/\sum f_i}$

$$= \left(\prod_{i=1}^{n} x_i^{f_i} \right)^{1/\Sigma f_i}$$

1. if all obs are same, then gm is same value.

2. GM of ratio of obs = ratio of their GM

ie $GM(x/y) = \dfrac{GM \text{ of } x}{GM \text{ of } y}$

Proof: $GM(x/y) = \left[\prod_{i=1}^{n} (x_i/y_i) \right]^{1/n}$

$$= \frac{\left(\prod_{i=1}^{n} x_i \right)^{1/n}}{\left(\prod_{i=1}^{n} y_i \right)^{1/n}}$$

$$= \frac{gm \text{ of } x}{gm \text{ of } y}$$ (proved)

3. log of GM is arithmetic mean of log of observation

ie $\boxed{\log(GM) = \frac{1}{n} \Sigma \log(x)}$

Harmonic Mean

(a) simple HM, $\bar{x}_h = \dfrac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$

$$= \frac{n}{\sum_{i=1}^{n} 1/x_i}$$

(b) weighted HM, $\bar{x}_h = \dfrac{\sum f_i}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \dots + \frac{f_n}{x_n}}$

$$\bar{x}_h = \frac{\sum_{i=1}^{n} f_i}{\sum_{i=1}^{n} \frac{f_i}{x_i}}$$

Properties of AM, GM and HM

① AM $\geq$ GM $\geq$ HM

② AM $\times$ HM $=$ (GM)^2

or, $GM = \sqrt{AM \times HM}$

Q. Find the missing frequencies in the following frequency distribution when AM = 11.09.

Class limits	9.3-9.7	9.8-10.2	10.3-10.7	10.8-11.2	11.3-11.7
frequency	2	5	f_3	f_4	14
Class limit	11.8-12.2	12.3-12.7	12.8-13.2	Total	
frequency	6	3	1	frequency = 60	

Solution

Class limits	Frequency (f)	Classmark (x)	xf
9.3-9.7	2	9.5	19
.	5	10	50
,	f_3	10.5	$10.5f_3$
,	f_4	11.	$11f_4$
,	14	11.5	161
!	6	12	72
.	3	12.5	37.5
12.8-13.2	1	13	13
	$N=\Sigma f=60$		

we have

Total frequency $\Sigma f = N = 60$

or, $31+f_3+f_4=60$

$$\boxed{f_3+f_4=29} \text{ ——— } (1)$$

And, AM $\bar{x} = \frac{1}{N}\Sigma x_i f_i$

$$11.09 = \frac{1}{60} \times (352.5 + 10.5 f_3 + 11 f_4)$$

$$11.09 \times 60 = 352.5 + 10.5 f_3 + 11 f_4$$

$$10.5 f_3 + 11 f_4 = 312.9 \quad \text{——— (2)}$$

$$11 f_3 + 11 f_4 = 319 \quad \text{— (3)}$$

Solving (2) and (3)

$$10.5 f_3 + \cancel{11 f_4} = 312.9$$

$$11 f_3 + \cancel{11 f_4} = 319.0$$

$$(-) \quad (-) \quad (-)$$

$$\cancel{-}0.5 f_3 = \cancel{-} 6.1$$

$$f_3 = \frac{6.1}{0.5} = \frac{61}{5} = 12.2 \approx 12$$

$$\therefore f_3 \approx 12$$

$$\therefore \boxed{f_4 = 29 - 12 = 17} \text{ (ans)}$$

$2u + 5x = 17$ $\qquad\qquad \bar{x} = 3$

or, $2u = 17 - 5x$

$$u = \frac{17 - 5x}{2}$$

$$u = \frac{17}{2} - \frac{5}{2}x$$

$$\bar{u} = \frac{17}{2} - \frac{5}{2}\bar{x}$$

$$\bar{u} = 17/2 - 5/2 \times 3 = \underline{\underline{\text{ans}}}$$