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$4\cos 36^\circ + \cot 7.5^\circ = \sqrt{n_1} + \sqrt{n_2} + \sqrt{n_3} + \sqrt{n_4} + \sqrt{n_5} + \sqrt{n_6}$

If $4\cos 36^\circ + \cot 7\frac{1}{2}^\circ = \sqrt{n_1} + \sqrt{n_2} + \sqrt{n_3} + \sqrt{n_4} + \sqrt{n_5} + \sqrt{n_6}$ where $n_i \in N$ then $\sum_{i=1}^{6} n_i^2$ equals

$\boxed{\cos 2\theta} = 1 - 2\sin^2\theta$

$\boxed{= 2\cos^2\theta - 1}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$.

$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}}$.

$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}}$

let $18^\circ = \theta$ $\quad 36^\circ = 2\theta \quad 5\theta = 90^\circ \quad 2\theta = 90^\circ - 3\theta$.

$\cos 2\theta = \cos(90 - 3\theta) = \sin 3\theta$

$\boxed{\cos 2\theta = 1 - 2\sin^2\theta}$.

$\boxed{\sin 3\theta} = \sin(2\theta + \theta) = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$= 2\sin\theta\cos^2\theta + (1 - 2\sin^2\theta)\sin\theta$

$= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$

$\boxed{= 3\sin\theta - 4\sin^3\theta}$

$1 - 2\sin^2\theta = 3\sin\theta - 4\sin^3\theta$

$4\sin^3\theta - 2\sin^2\theta - 3\sin\theta + 1 = 0$

$4\sin^3\theta - 4\sin^2\theta + 2\sin^2\theta - 2\sin\theta - \sin\theta + 1 = 0$

$4\sin^2\theta(\sin\theta - 1) + 2\sin\theta(\sin\theta - 1) - 1(\sin\theta - 1) = 0$

$(\sin\theta - 1)(4\sin^2\theta + 2\sin\theta - 1) = 0$

$4\sin^2\theta + 2\sin\theta - 1 = 0 \Rightarrow \boxed{\sin\theta} = \frac{-2 + \sqrt{4 + 16}}{8} = \frac{-2 + 2\sqrt{5}}{8} = \boxed{\frac{\sqrt{5} - 1}{4}}$

$\cos 2\theta = 1 - 2\sin^2\theta \Rightarrow \cos 36^\circ = 1 - 2\sin^2 18^\circ = 1 - \cancel{2} \times \frac{(\sqrt{5} - 1)^2}{\cancel{16}\,8} = 1 - \left(\frac{6 - 2\sqrt{5}}{8}\right) = 1 - \left(\frac{3 - \sqrt{5}}{4}\right)$

$\cos 36^\circ = \left(\frac{\sqrt{5} + 1}{4}\right) \Rightarrow \boxed{4\cos 36^\circ = \sqrt{5} + 1}$

$\cot\theta = \frac{\cos\theta}{\sin\theta} = \sqrt{\frac{1 + \cos 2\theta}{1 - \cos 2\theta}} \qquad \theta = 7.5^\circ \qquad 2\theta = 15^\circ$.

$\cos 15^\circ = \cos(45 - 30) = \cos 45\cos 30 + \sin 45\sin 30 = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2} = \frac{\sqrt{3} + 1}{2\sqrt{2}}$

$\cot 7.5^\circ = \sqrt{\frac{2\sqrt{2} + \sqrt{3} + 1}{2\sqrt{2} - \sqrt{3} - 1}} = \sqrt{\frac{(2\sqrt{2} + \sqrt{3} + 1)^2}{(2\sqrt{2})^2 - (\sqrt{3} + 1)^2}} = \frac{2\sqrt{2} + \sqrt{3} + 1}{\sqrt{8 - (4 + 2\sqrt{3})}} = \frac{2\sqrt{2} + \sqrt{3} + 1}{\sqrt{4 - 2\sqrt{3}}}$

$= \frac{2\sqrt{2} + \sqrt{3} + 1}{\sqrt{(\sqrt{3} - 1)^2}} = \frac{2\sqrt{2} + \sqrt{3} + 1}{\sqrt{3} - 1} = \frac{(2\sqrt{2} + \sqrt{3} + 1)(\sqrt{3} + 1)}{(\sqrt{3})^2 - 1^2} = \frac{2\sqrt{6} + 2\sqrt{2} + 3 + \sqrt{3} + \sqrt{3} + 1}{2}$

$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$.

$= \frac{2\sqrt{6} + 2\sqrt{3} + 2\sqrt{2} + 4}{2} = \sqrt{6} + \sqrt{3} + \sqrt{2} + 2 = \boxed{\sqrt{6} + \sqrt{3} + \sqrt{2} + \sqrt{4}}$

$4\cos 36^\circ + \cot 7.5^\circ = \sqrt{5} + 1 + \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$

$= \sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{6}$.

$\sum_{i=1}^{6} n_i^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = \frac{6\cdot 7\cdot 13}{6} = 91$

If sum of solution(s) in $[0, 3\pi]$ of the trigonometric equation $\operatorname{cosec}\theta = 1 + \cot\theta$ is $k\pi$, then value of k is

(A) 1 (B) 2 (C) 3 (D) 5

C

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$. $\rightarrow \boxed{\operatorname{cosec}\theta = 1 + \cot\theta}$

If $\sin\theta = 0 \rightarrow$ $\operatorname{cosec}\theta$ is undefined

$\frac{1}{\sin\theta} = 1 + \frac{\cos\theta}{\sin\theta}$

$\sin\theta + \cos\theta = 1$

$\downarrow$

$\cancel{\sin^2\theta} + \cancel{\cos^2\theta} + 2\cos\theta\sin\theta = \cancel{1}$

$\sin 2\theta = 0 = \sin n\pi$

$2\cos\theta\sin\theta = 0$

$\cos\theta = 0$

$\frac{(4n+1)\pi}{2}$. $\quad \sin = 1 \quad \cos = 0$

odd $(2n+1)\pi$ $\quad \sin = 0$.

$2n\pi$ (even multiples) $\quad \sin = 0$.

$\sin x = 0$.

$x = n\pi$

$(2n+1)\pi$ — $\sin=0$.

$\cos=0$. $\sin=-1$. $(4n+3)\frac{\pi}{2}$.

$2n\pi$ (even multiples) $\sin=0$.

$x=n\pi$

$\theta = \cancel{0}, \frac{\pi}{2}, \cancel{\pi}, \frac{3\pi}{2}, \cancel{2\pi}, \frac{5\pi}{2}, \cancel{3\pi}$.

$2\theta = n\pi$

$\theta = n\frac{\pi}{2}$

Sum of all $\theta = 10.5\pi$. $2+2.5 = 4.5\pi$

The number of solution of the equation sin x + cos x = 2 is 4

$\sin x + \cos x = 2$.

$$a\cos x + b\sin x = \sqrt{a^2+b^2}\left[\underbrace{\frac{a}{\sqrt{a^2+b^2}}}_{\sin\alpha}\cos x + \underbrace{\frac{b}{\sqrt{a^2+b^2}}}_{\cos\alpha}\sin x\right]$$

$$\frac{a}{\sqrt{a^2+b^2}} = \sin\alpha. \quad \sin^2\alpha = \frac{a^2}{a^2+b^2} \quad \cos^2\alpha = 1-\sin^2\alpha = 1-\left(\frac{a^2}{a^2+b^2}\right)$$

$$\cos^2\alpha = \frac{b^2}{a^2+b^2}$$

$$\cos\alpha = \frac{b}{\sqrt{a^2+b^2}}$$

$$a\cos x + b\sin x = \sqrt{a^2+b^2}\left(\sin\alpha\cos x + \cos\alpha\sin x\right)$$

$$= \sqrt{a^2+b^2}\,\boxed{\sin(\alpha+x)}$$

$$-1 \leq \sin\theta \leq 1$$

$$-1 \leq \sin(\alpha+x) \leq 1$$

$$-\sqrt{a^2+b^2} \leq \sqrt{a^2+b^2}\sin(\alpha+x) \leq \sqrt{a^2+b^2}$$

$$-\sqrt{2} \leq \sin x + \cos x \leq \sqrt{2}$$

$$\sin x + \cos x \neq 2.$$

$$-\sqrt{a^2+b^2} \le \sqrt{a^2+b^2}\sin(\alpha+x) \le \sqrt{a^2+b^2}$$

$-\sqrt{2} \le$

$$\boxed{-\sqrt{a^2+b^2} \le a\cos x + b\sin x \le \sqrt{a^2+b^2}}$$

$\sin x + \cos x \ne 2.$

Number of integral values of k for which the equation $(3\sin x + 4\cos x + 4)^2 = 9k^2$ has a solution, is

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$$\boxed{-\sqrt{a^2+b^2} \le a\sin x + b\cos x \le \sqrt{a^2+b^2}}$$

$$(3\sin x + 4\cos x + 4)^2 = 9k^2$$

$$3\sin x + 4\cos x + 4 = \pm 3k.$$

$$3\sin x + 4\cos x = -4 \pm 3k.$$

$$-\sqrt{3^2+4^2} \le 3\sin x + 4\cos x \le \sqrt{3^2+4^2}$$

$$-5 \le 3\sin x + 4\cos x \le 5$$

$$-5 \le -4 \pm 3k \le 5$$

$$-5 \le -4 + 3k \le 5$$

$3k \ge -1$ $\quad$ $3k \le 9$

$k \ge -1/3.$ $\quad$ $k \le 3.$

$$\boxed{-\frac{1}{3} \le k \le 3}$$

$\Downarrow$

$k = 0, 1, 2, 3$

$$-5 \le -4 - 3k \le 5.$$

$3k \le 1$ $\quad$ $3k \ge -9$

$k \le \frac{1}{3}$ $\quad$ $k \ge -3.$

$$\boxed{-3 \le k \le 1/3}$$

$k = -3, -2, -1, 0$

The number of solution of the equation $\cos^2 x + \sin x = \frac{11}{9}$ in $x \in [-2\pi, 2\pi]$, is

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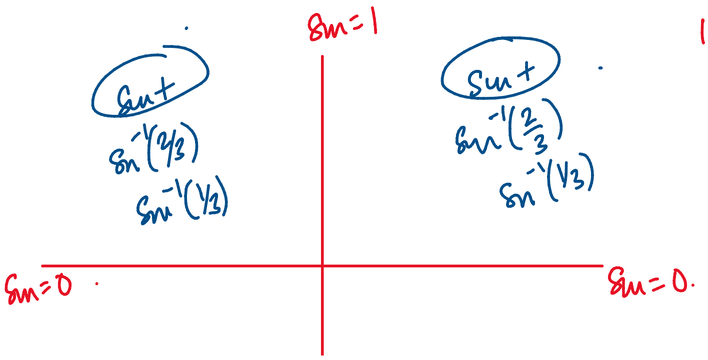

$$\cos^2 x + \sin x = \frac{11}{9}$$

$$1 - \sin^2 x + \sin x = \frac{11}{9}$$

$$\sin^2 x - \sin x + \frac{2}{9} = 0.$$

$$9\sin^2 x - 9\sin x + 2 = 0$$

$$9\sin^2 x - 6\sin x - 3\sin x + 2 = 0.$$

$$3\sin x(3\sin x - 2) - 1(3\sin x - 2) = 0$$

$\sin\theta = \frac{1}{2}$

$\theta = 30^\circ, 150^\circ$

$\sin^{-1}(1/2)$

$\sin = 0$.

$\sin = 0$.

$9\sin^2 x - 6\sin x$ …

$3\sin x(3\sin x - 2) - 1(3\sin x - 2) = 0$

$(3\sin x - 2)(3\sin x - 1) = 0$.

$\sin x = \frac{2}{3}, \frac{1}{3}$.