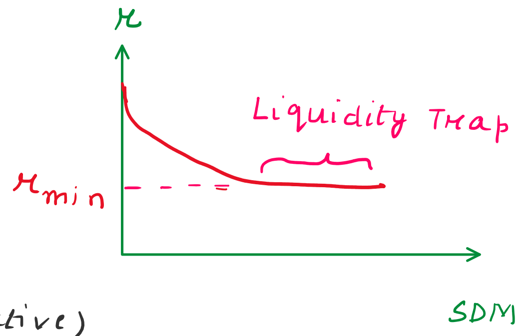
Special case:

If $SDM \to \infty$, $\left.\frac{dr}{dY}\right|_{LM} = -\frac{L_Y}{L_r} = -\frac{L_Y}{\infty} \to 0$. [Horizontal LM].

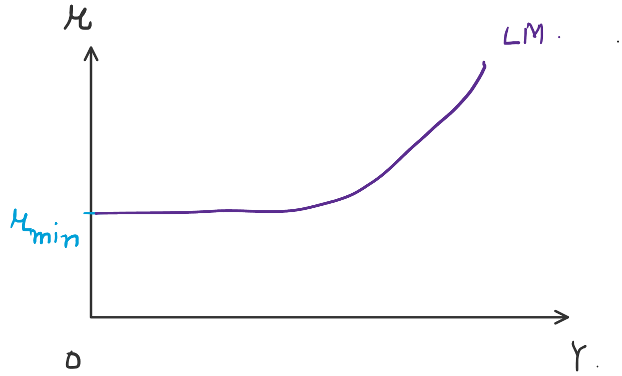


$SDM = g(r)$, $g' < 0$.

$r \downarrow \Rightarrow SDM \uparrow \Rightarrow$ After a pt $r = r_{min}$, $SDM \to \infty$, as a result everyone will want to hold (speculative) money and no one will want hold asset. This situation is known as liquidity trap. Essentially in the case, everyone wants to sells to assets to hold money and there will no buyer of the asset. (Asset mkt fails).

$\therefore \quad SDM = g(r), \; g' < 0, \; r_{min} < r < r_{max}$

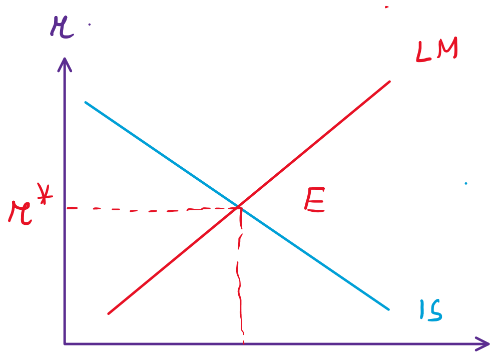
[Since assets markets are very vulnerable].



Equillibrium in the IS-LM Model:

IS: $Y = C(Y-T) + I(r) + G$.

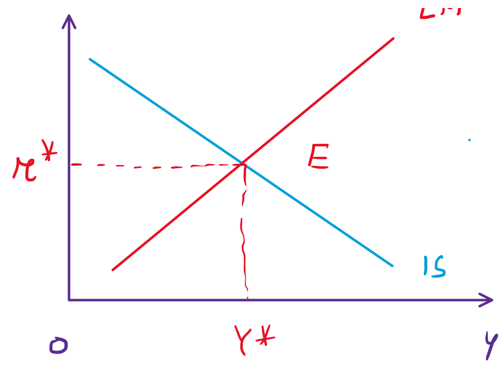
LM: $\frac{\overline{M}}{P} = L(Y, r)$



IS-LM Model:

IS: $Y = C(Y-T) + I(r) + G$

LM: $\frac{\bar{M}}{P} = L(Y, r)$



Eg: $C = \bar{C} + c'(Y-T)$, $0 < c' < 1$, $\bar{c} > 0$.

$T = t \cdot Y$, $0 < t < 1$.

$I = \bar{I} - br$, $b > 0$, $\bar{I} > 0$.

$G = \bar{G}$

$M_s = \bar{M}$

$L(Y, r) = kY - hr$, $k, h > 0$

Find the IS-LM Equillibrium.

$\left|\frac{\partial I}{\partial r}\right| = b$ [ΔI due to Δr]

[Interest sensitivity of inv]

$\frac{\partial L}{\partial Y} = k$, $\left|\frac{\partial L}{\partial r}\right| = h$.

[Income sensitivity of M_d]

[Interest sensivity of M_d]

IS: $Y = C + I + G$

$Y = \bar{C} + c'(Y - t \cdot Y) + \bar{I} - br + \bar{G}$

$[1 - c'(1-t)]Y + br = (\bar{C} + \bar{I} + \bar{G})$ ---- (i)

LM: $k \cdot Y - hr = \frac{\bar{M}}{P}$ .... (ii)

solve for Y, r

Using the Cramer's Rule:

$$Y^* = \frac{\begin{vmatrix} (\bar{C} + \bar{I} + \bar{G}) & b \\ \bar{M}/P & -h \end{vmatrix}}{\begin{vmatrix} [1 - c'(1-t)] & b \\ k & -h \end{vmatrix}}$$

$$r^* = \frac{\begin{vmatrix} [1 - c'(1-t)] & (\bar{C} + \bar{I} + \bar{G}) \\ k & \bar{M}/P \end{vmatrix}}{\begin{vmatrix} [1 - c'(1-t)] & b \\ k & -h \end{vmatrix}}$$

$$Y^* = \frac{\left(-h(\bar{C} + \bar{I} + \bar{G}) - b \cdot \bar{M}/P\right) / -h}{\left(-h[1 - c'(1-t)] - b \cdot k\right) \div -h} = \frac{(\bar{C} + \bar{I} + \bar{G}) + \frac{b}{h} \cdot \frac{\bar{M}}{P}}{\underbrace{[1 - c'(1-t)]}_{>0} + \underbrace{b\left(\frac{k}{h}\right)}_{>0}} > 0.$$

$$r^* = \frac{\left(\frac{\bar{M}}{P}[1 - c'(1-t)] - k \cdot (\bar{C} + \bar{I} + \bar{G})\right) / -h}{\left(-h[1 - c'(1-t)] - bk\right) \div -h} = \frac{\frac{k}{h}(\bar{C} + \bar{I} + \bar{G}) - \frac{1}{h} \cdot \frac{\bar{M}}{P}[1 - c'(1-t)]}{[1 - c'(1-t)] + b\left(\frac{k}{h}\right)}$$

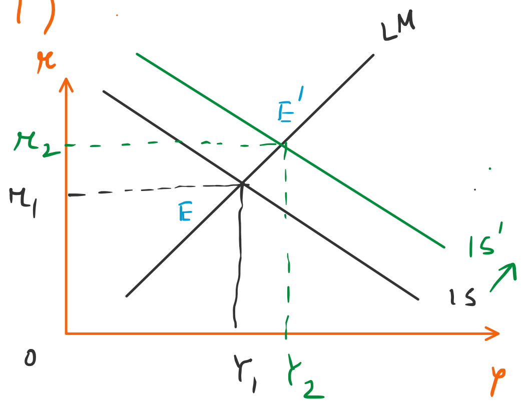
$$r^* = \frac{(\;[\text{illegible}]\;)/-h}{\left(-h[1-c'(1-t)] - bk\right)\cdot -h} = \frac{\frac{k}{h}(\bar{C}+\bar{I}+G) - \frac{1}{h}\cdot\frac{\bar{M}}{P}[1-c'(1-t)]}{[1-c'(1-t)] + b\cdot\left(\frac{k}{h}\right)}.$$

$$r^* > 0 \quad \text{iff} \quad \frac{k}{h}(\bar{C}+\bar{I}+G) > \frac{1}{h}\cdot\frac{\bar{M}}{P}[1-c'(1-t)].$$

Counter-recessionary policies in IS-LM:

(i) Govt expenditure multiplier ($G\uparrow$)

Output $\uparrow$ ($Y_1 \to Y_2$), Interest rate $\uparrow$ ($r_1 \to r_2$)



$$\frac{\partial Y^*}{\partial G} = \frac{1}{[1-c'(1-t)] + b\cdot\frac{k}{h}} > 0 \text{ but } \gtrless 1.$$

$$\frac{\partial r^*}{\partial G} = \frac{k/h}{[1-c'(1-t)] + b\cdot\frac{k}{h}} > 0.$$

Recap: SKM Govt Exp Multiplier: $\frac{dY}{dG} = \frac{1}{1-c'(1-t)}$

i.e $\left.\frac{dY}{dG}\right|_{SKM} > \left.\frac{\partial Y}{\partial G}\right|_{IS-LM}$ (Govt exp multiplier is weaker in IS-LM).

Q. Why?

$G\uparrow \Rightarrow AD\uparrow \xrightarrow{} (1) \Rightarrow Y\uparrow$ (same as SKM explanation)

$\hookrightarrow$ TDM $\uparrow$ $\Rightarrow$ Money mkt equi: $\left(\frac{\bar{M}}{P}\right) = \overset{\uparrow}{TDM} + \overset{\downarrow}{SDM}$.

$\Rightarrow SDM\downarrow \Rightarrow r\uparrow \Rightarrow I\downarrow \Rightarrow AD\downarrow \to (II)$

Assuming that the primary impact on AD dominates the second, increase in r, partly "crowds out" private investment, hence IS-LM govt exp multiplier is weaker than the SKM.

Note: Evaluate Govt exp Multiplier [Find change in Y, r due to G]

IS: $[1-c'(1-t)]Y + br = (\bar{C} + \bar{I} + \bar{G})$ --- (i)

LM: $k.Y - hr = \frac{\bar{M}}{P}$ ---- (ii)

Diff: IS: $[1-c'(1-t)].dY + b.dr = dG$

LM: $k.dY - h.dr = 0.$

$$dY = \frac{\begin{vmatrix} dG & b \\ 0 & -h \end{vmatrix}}{\begin{vmatrix} [1-c'(1-t)] & b \\ k & -h \end{vmatrix}} = \frac{(-h.dG)/-h}{([1-c'(1-t)](-h) \,\cancel{-}\, bk)/-h}$$

$$\frac{dY}{dG} = \frac{1}{[1-c'(1-t)] + b.\frac{k}{h}}$$